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Essays on Liquidity, Banking, and Monetary Policy
by
Jaevin Park

A dissertation presented to the
Graduate School of Arts and Sciences
of Washington University in
partial fulfillment of the
requirements for the degree
of Doctor of Philosophy

May 2016
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Jaevin Park

Washington University in St. Louis

May 2016

Dedicated to my parents and wife, hyerim.

ABSTRACT OF THE DISSERTATION

Essays on Liquidity, Banking, and Monetary Policy

by

Jaevin Park

Doctor of Philosophy in Economics

Washington University in St. Louis, 2016

Professor Stephen D. Williamson, Chair

The first chapter develops a new theory of bank capital requirements. A general equilibrium banking model is constructed in which deposit claims backed by bank assets support secured credit arrangements with limited commitment. Bank capital, a contingent claim on bank assets, is costly to hold when the value of assets is insufficient to support an efficient credit arrangement. However, if there is non-diversifiable aggregate risk, requiring banks to hold additional bank capital in the high-return state can be beneficial since it can relax the limited commitment constraint in the low-return state by affecting asset prices. Thus bank capital requirements can improve economic welfare by trading off the opportunity cost of holding additional bank capital for the benefit from sharing consumption risk.

The second chapter contains a study of how private information can restrict liquidity insurance and the implementation of monetary policy. Lack of record-keeping implies that recognizable assets are essential for trade and also generates a private information problem when agents are subject to idiosyncratic liquidity shocks. A banking arrangement with self-selection that improves liquidity provision through the use of two different liquid assets is considered. It is found that when the incentive constraint binds with asset scarcity, there exists a liquidity premium on illiquid assets which reveals private information. The model is extended to include monetary policy, specifically Open-Market-Operations. It is shown that liquidity trap can exist when truth-telling incentive constraints bind.

Chapter 1

Aggregate Risk, Inside Money, and Bank Capital Requirements

1.1 Introduction

I study how bank capital requirements can influence credit cycles in a liquidity perspective. Repullo and Suarez (2013) show that the risk-based capital requirements can amplify credit cycles because higher capital levels are needed in the recessions. However, it is also important to consider not only the risk aspect, but also the liquidity aspect of bank capital requirements because bank liabilities such as deposit claims and bank notes are useful for transactions as collateral while bank capital is not. If bank capital is not useful for credit arrangements of depositors, raising capital requirements can just reduce credit availability of individual depositors given that the supply of bank assets is fixed. It is because a proportion of assets for bank capital holder, indicated as a red rectangle in the Figure 1.1, cannot be a claim for depositors while the rest of assets, a blue rectangle, can support the deposit claim. It implies that bank capital requirements can adjust the pledgeability of bank assets, i.e. a proportion of assets that serves as collateral, under limited commitment of banks.

This paper develops a novel mechanism by which bank capital requirements can improve

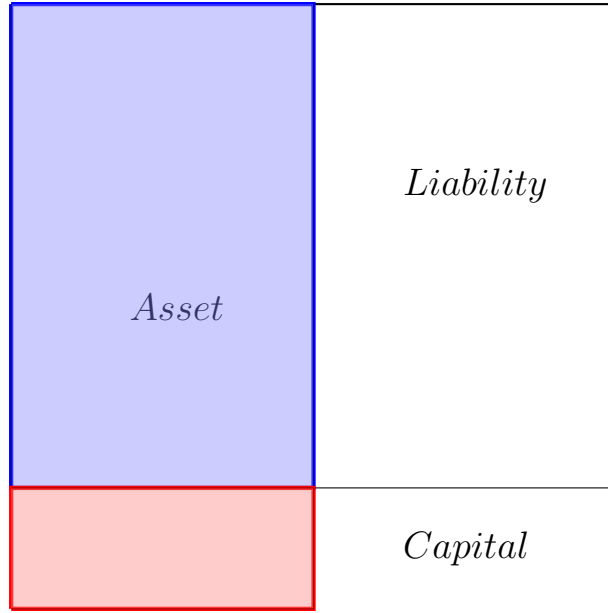


Figure 1.1: Bank balance sheet with liquidity difference

economic welfare by promoting efficient liquidity provision across the states. In particular, given aggregate risk and limited commitment, state-contingent bank capital requirements can play a role in sharing liquidity risk by adjusting the pledgeability of bank assets. This role of bank capital requirements can provide insight into credit-cycle stabilization and so-called macro-prudential policy.

In order to explore this issue I develop an asset-exchange model in which bank liabilities are used to facilitate payments and settlement in an explicit way. This micro-founded model has the advantage of easily incorporating informational frictions such as limited commitment and imperfect memory. It is also highly tractable, with an array of assets and a contingent form of banking contract. This framework is also suitable for welfare analysis in a general equilibrium as the cost of holding bank capital is determined endogenously in the model without externalities. The basic structure of the model comes from Rocheteau and Wright (2005): in the model *ex ante* heterogeneous agents can trade in the decentralized meetings and their asset portfolios are rebalanced in the centralized markets. The structure of banking arrangement is borrowed from Williamson (2012), where bank liabilities are protected only

by the value of bank assets with limited commitment. There is a fixed supply of private assets for which the returns are subject to aggregate risk. Given the aggregate risk, a contingent banking contract is considered to maximize the *ex ante* expected value of depositors under perfect competition.

Limited commitment is a key element in the model, as it can restrict credit provision by banks.¹ Since assets are useful for supporting these credit arrangements, the price of the assets can be valued not only for their expected stream of future yields, but also for the usefulness in exchange. This gives rise to a liquidity premium in the price of assets in equilibrium. In equilibrium under perfect competition banks would not hold bank capital voluntarily when the supply of real assets is insufficient to support credit arrangements of the depositors. However, this competitive equilibrium allocation can be constrained-suboptimal according to the result of Geanakoplos and Polemarchakis (1986).² This is because the asset market is incomplete when there is a non-diversifiable aggregate risk in the return of assets.³ Thus there is a possibility to improve economic welfare manipulating the degree of limited commitment, although the contract is complete and there is no externality.

This paper shows that pro-cyclical capital requirements can improve welfare by stabilizing credit cycles. Requiring additional bank capital just reduces the pledgeability of assets so that secured credit is constrained in the high return states. However, restricting the pledgeability of assets in the high-return states can affect *ex ante* asset prices because the liquidity premium on the assets, which is associated with trade inefficiency in each state, will change. Then the consumption level in the low-return state can increase since the limited commitment constraint is relaxed as the asset price rises. Thus there is a trade-off between the opportunity cost of holding additional bank capital and the benefit from sharing liquidity risk. It is shown that imposing a bank capital requirement in the high-return state

¹Unlike government debt, which is supported by the commitment of taxation, bank liabilities are only protected by the collateralized assets under the limited commitment of banks.

²Geanakoplos and Polemarchakis (1986) show that the equilibrium allocation is constrained suboptimal in a model of competitive general equilibrium with incomplete markets.

³If the asset market is complete then the equilibrium allocation is constrained optimal even though there exists aggregate risk as shown in Kehoe and Levine (1993).

can improve welfare as much as the depositors are risk-averse.

This paper makes some key contributions. The mechanism of the main result is different from the path in the previous literature on pro-cyclical bank capital requirements which is based on systemic risk. For example, the counter-cyclical buffer in the Basel III accord, which requires additional bank capital in a period of excess credit growth, is proposed to reduce a social cost associated with default of banks in recessions. Thus bank capital is accumulated in the high-return states to be used as a buffer in the low-return states. In this paper capital requirements can transfer credit availability or purchasing power from the high-return state to the low-return state by affecting asset prices without a real transfer. Thus the same pro-cyclical capital requirement is beneficial for society, but in this paper it is beneficial because this pro-cyclical requirement can stabilize credit cycles.

This result provides a new rationale for bank capital requirements. A conventional rationale for bank capital requirements is based on deposit insurance: banks will tend to take too much risk under this safety net, so that bank capital requirements are needed to correct the moral hazard problem created by deposit insurance. Alternatively, it is sometimes argued that bank capital requirements can be justified based on an externality associated with systemic risk. For example, contagion can justify government interventions since a default of one bank could lead to a chain reaction where many other financial intermediaries could go bankrupt. However, this paper shows that capital requirements can be rationalized by incomplete market and limited commitment.

There exists an equilibrium in which bank capital can be held voluntarily even though it is costly to hold. Since bank capital is not useful for exchange, it is costly for a bank to hold assets to support bank capital when asset prices reflect a liquidity premium. However, if the assets are plentiful only in the high-return state, the *ex post* marginal benefit of holding assets will be less than the *ex ante* marginal cost of buying assets. Then bank capital is useful in the view of depositors because they can avoid holding unnecessary assets in the high-return state. This result can provide an alternative explanation for the historical fact

that banks have at times held capital in excess of capital requirements. Berger, Herring, and Szego (1995) report that in the 1840s U.S. commercial banks had equity-to-asset ratios of over 50 percent. This ratio declined over time, but it has been kept above the required level even after the Basel I capital requirement was imposed in the 1990s. The excessive holding of bank capital has brought about theories for another role of bank capital. For example, Diamond and Rajan (2000) present a model in which voluntarily held bank capital serves as a buffer in recessions to prevent bank runs. In this paper I show that strictly positive bank capital can exist in equilibrium without additional functions of bank capital.

Finally, bank capital requirements can influence macroeconomic variables and the implementation of monetary policy. In order to study this issue I extend the model by introducing two additional government-issued assets, i.e. money and government bonds. In the full-fledged model, the real value of outstanding government debts is kept as constant and the central bank chooses the proportion between money and government bonds through open market operations as shown in Williamson (2014). There is an idiosyncratic shock faced by depositors under which one type of depositor must use currency for trade and the other type can make a credit arrangement with government bonds and private assets. Since bank capital requirements can affect the liquidity premium on the asset prices, real interest rates on the assets can be adjusted without using open market operations. This path allows us to consider bank capital requirements as an unconventional monetary policy tool at the zero-lower-bound where conventional monetary policy is limited. Given a fixed monetary policy, imposing bank capital requirements can reduce the feasible set of equilibrium allocations. If the liquidity premium on the backed assets rises by imposing bank capital requirements, the inflation rate must also rise in equilibrium to make rates of return on currency and government bonds equal. Thus given the same credit arrangements, the amount of currency trade can decrease by imposing bank capital requirements.

1.1.1 Related Literature

This paper is related to the literature that studies the necessity of bank capital regulations in a theoretical way.⁴ One strand of the literature focuses on the moral hazard of banks induced by deposit insurance. For example, Kareken and Wallace (1978) show that deposit insurance can create the moral hazard of banks. Kim and Santomero (1988) and Furlong and Keeley (1989) conduct a pioneering study on the optimal risk-taking problem of banks by using a mean-variance model and by considering the option value of deposit insurance, respectively. Dewatripont and Tirole (2012) and Boyd and Hakenes (2014) concentrate more on managerial looting incentive than risk-taking behavior. The other strand of the literature is based on the externality associated with systemic risk. In this strand, Allen and Gale (2006) study an environment where credit risk is not sufficiently transferred to the insurance institutions to reduce systemic risk. Goodhart et. al. (2012) explore various types of financial regulations to control fire sales. Farhi and Tirole (2012) build a model to analyze leverage and maturity-mismatch to address the optimal macro-prudential policy. Lorenzoni (2008) and Jeanne and Korinek (2011) emphasize pecuniary externality in which banks cannot internalize systemic risk in the incomplete market. Farhi and Werning (2015) consider aggregate demand externalities generated by nominal rigidities additionally along with the pecuniary externalities associated with market incompleteness which is same as this paper.

In this paper I focus on limited commitment instead of asymmetric information and externality to rationalize bank capital requirements. This limited commitment friction is introduced by Gertler and Kiyotaki (2013) to explain that bank capital, i.e. net worth, is helpful to raise funds from depositors when private banks are not trustworthy. Williamson (2014) develops this idea to determine the bank capital structure endogenously in his model. In those papers bank capital can adjust the pledgeability and liquidity of assets. However, the rationale for capital requirements is first studied in the present paper.

⁴See also VanHoose (2007) for a literature review on banking theories with the bank capital regulations.

The function of bank liabilities as a means of payment and settlement is also considered with capital regulations in the previous literature. For example, Begenau (2015) shows that bank capital requirements can, in fact, increase bank lending because the reduced supply of bank liabilities adjusts the interest rate downwards. However, in Begenau (2015) bank capital is held only when capital requirements are enforced. While in this paper bank capital can be held voluntarily without bank capital requirements.

This paper builds on the literature that provides micro-foundations for monetary economics as pioneered by Kiyotaki and Wright (1989) and Lagos and Wright (2005). Banking models with explicit trade frictions are developed by Freeman (1988), Champ, Smith and Williamson (1996) and Sanches and Williamson (2010). The role of assets in exchange is studied by Geromichalos, Licari and Suarez-Lledo (2007), Lagos and Rocheteau (2008). Limited commitment in assets-exchange is studied by Kiyotaki and Moore (2005) and Venkateswaren and Wright (2013). Aggregate risk in the return of assets is introduced in Lagos (2010) to explain the equity-premium puzzle and in Andolfatto, Berentsen and Waller (2014) to consider the optimal information disclosure. Bank capital is recognized as a non-pledgeable part of assets in Williamson (2014). But the rationale for bank capital requirements is considered in the present paper.

The organization of the paper is as follows. In the second section I describe the elements of the model. In the third section a simple model with one risky asset is characterized and analyzed with bank capital requirements. I introduce money and government bonds in the fourth section to consider the relationship between bank capital requirements and monetary policy. The final section concludes.

1.2 Model

The model structure is based on Rocheteau and Wright (2005). Time $t = 0, 1, 2, \dots$ is discrete and the horizon is infinite. Each period is divided into two sub-periods - the centralized

market (*CM*) followed by the decentralized market (*DM*). There is a continuum of buyers, sellers and bankers, each with unit mass. An individual buyer has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)],$$

where $H_t \in \mathbb{R}$ is labor supply in the *CM*, $x_t \in \mathbb{R}_+$ is consumption in the *DM*, and $0 < \beta < 1$. Assume that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u'(0) = \infty$, $u'(\infty) = 0$, and $-x \frac{u''(x)}{u'(x)} = \gamma < 1$.⁵ Each seller has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t],$$

where $X_t \in \mathbb{R}$ is consumption in the *CM*, and $h_t \in \mathbb{R}_+$ is labor supply in the *DM*. An individual banker has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - H_t],$$

where $H_t \in \mathbb{R}_+$ is labor supply in the *CM*, $X_t \in \mathbb{R}_+$ is consumption in the *CM*.

Buyers can produce in the *CM*, but not in the *DM* while sellers can produce in the *DM*, but not in the *CM*. Bankers can produce and consume in the *CM*, but cannot participate in the *DM*. One unit of labor input produces one unit of perishable consumption good either in the *CM* or in the *DM*.

In this economy there are two kinds of public assets, fiat money and one-period government bonds, issued by the fiscal authority. Fiat money trades at price ϕ_t in terms of goods in the *CM* of period t . One-period maturity government bonds, which are obligations to pay one unit of fiat money in the *CM* of period $t + 1$, sell at price z_t in terms of goods in the *CM* of period t . There also exists one private asset - a divisible Lucas tree. It is endowed to buyers in the *CM* of the initial period $t = 0$ with a fixed unit supply. The Lucas tree pays

⁵Constant relative risk aversion is useful to derive the benefit of consumption risk-sharing explicitly in the model. It is also useful to have a unique equilibrium because the demand for assets is strictly increasing in rates of return so that substitution effects dominate income effects when $\gamma < 1$.

off y_t units of consumption goods as a dividend and trades at the price ψ_t in terms of goods in the *CM* of period t . The dividend of the Lucas tree, y_t , is an *i.i.d* random variable which can take on two possible values, $0 < y^l \leq y^h < \infty$. Let π denote the probability of a high dividend y^h , and let $\bar{y} \equiv \pi y^h + (1 - \pi)y^l$ as an expected payoff of this random dividend.

In the beginning of the period t *CM*, all agents meet and debts or obligations are paid off. Buyers receive lump-sum transfer (or pay lump-sum tax) and the holders of the Lucas tree receive the dividends. Then a Walrasian market opens, goods are produced, assets are traded and buyers deposit goods or assets into a banker with a contingent deposit contract. The asset market is closed and the next period $t + 1$ dividend of the Lucas tree is known in the end of the period t *CM*.

In the *DM* each buyer meets each seller bilaterally and the terms of trade are determined by bargaining. The buyer makes a take-it-or-leave-it offer to the seller. There is no record-keeping technology in the *DM* so that agents are anonymous. Limited commitment is assumed so that no one can be forced to work. Thus no unsecured credit is available, recognizable assets are essential for trade, and trade must be *quid pro quo*.

Similar to Sanches and Williamson (2010), there are two kinds of random matches in the *DM*. In a fraction ρ of non-monitored *DM* meetings fiat money is only recognized by sellers. In $1 - \rho$ fraction of monitored *DM* meetings the entire asset portfolio held by the buyer can be verified by the seller so that a secured credit arrangement is available for trade. I assume that fiat money, i.e. currency, is portable and can be used on the spot in the *DM* while the other assets are not.⁶ Thus deposit claims backed by the assets can be used on the spot to transfer account balances of the buyer to the seller in the monitored *DM* meetings. Since deposit claims issued by buyers or sellers can violate no record-keeping environment in the *DM*, I assume that a representative banker provides a banking arrangement by issuing deposit claims.⁷ Note that perfect competition is assumed among the bankers so that a

⁶Even if buyers can use their asset holdings directly for the trade, there is no more benefit from the direct asset-trade because a banker provides the optimal arrangement for buyers with zero profit.

⁷Since bankers have a linear utility function the same as buyers and sellers in the *CM*, there is no more advantage for using deposit claims of a banker than deposit claims of the other agents.

banker suggests a deposit contract that provides the maximum expected value of depositors.

Given no memory and limited commitment, the banker can abscond in the next *CM*, but the backed assets would be seized and transferred to the seller.⁸ Thus the asset portfolio except for currency can be pledged as collateral as shown in Kiyotaki and Moore (2005) or Venkateswaran and Wright (2013). One difference from their models is that the pledgeability of the assets can be chosen by imposing contingent bank capital requirements. Thus when a representative banker offers a contingent deposit contract, in which the payoff of deposit claims can vary by states, a proportion of the assets which backs the deposit claims can be adjusted by imposing bank capital requirements.⁹

When the contract term is arranged buyers do not know what types of meeting they will be in during the next *DM*. Thus the banking contract also provides liquidity insurance as shown in Diamond and Dybvig (1983). Assume that the size of shock ρ is exactly observable and type is public information. Thus I can set aside the bank runs issue. After type is realized, type 1 buyers who will move to ρ non-monitored meetings can withdraw currency from the banker when they meet the banker. Type 2 buyers who will move to $1 - \rho$ monitored meetings remain with deposit claims. To support the banking arrangement I assume that the buyer can meet only one banker in the *CM* after their liquidity shock is realized.¹⁰

The timing is as follows. In the beginning of *CM* debts are paid off and all buyers provide labor and trade assets and write a contract with a banker in a Walrasian market. After liquidity shock is realized buyers learn their type and ρ buyers meet the banker to withdraw money. In the end of *CM* the dividend for the next period is known for everyone. In the *DM* buyers meet sellers randomly in the bilateral meeting and make take-it-or-leave-it offers. In the next *CM* $1 - \rho$ sellers can receive *CM* goods by redeeming deposit claims to the banker or sell them to buyers.

⁸All agents are subject to the same degree of limited commitment.

⁹The contract term must be state-contingent because no one knows the aggregate state when the contract is written.

¹⁰Note that if *ex post* asset-trading among buyers is allowed then the banking contract is unraveled and collapsed as shown in Jacklin (1987).

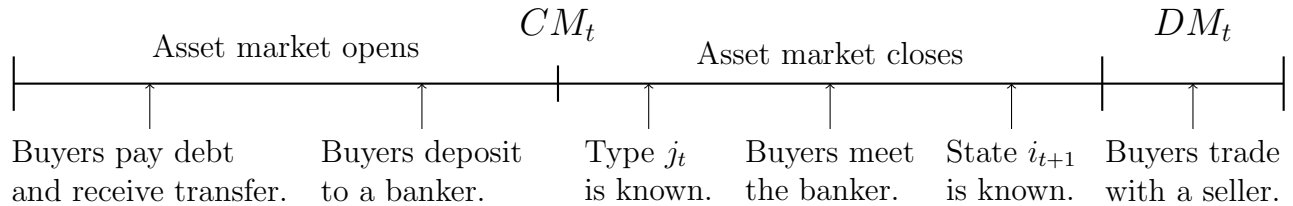


Figure 1.2: Time line

1.2.1 Government

In the model the consolidated government consists of the fiscal authority and central bank. The fiscal authority issues one-period nominal government bonds in the CM and pays interests in the next CM . The monetary authority issues fiat money and injects (or absorbs) fiat money in the markets by exchanging fiat money with government bonds, i.e. open market operations. In addition, the fiscal authority can collect a lump-sum tax from buyers (or provide a transfer to buyers) in the CM .¹¹ In period $t = 0$ government bonds are issued and fiat money is injected with lump-sum transfer, τ_0 , and in the following periods outstanding fiat money and government bonds are supported by tax or transfer over time. So the consolidated government budget constraint for $t = 0$ is

$$\phi_0(M_0 + z_0 B_0) = \tau_0,$$

and for $t = 1, 2, 3, \dots$

$$\phi_t\{M_t - M_{t-1} + z_t B_t - B_{t-1}\} = \tau_t$$

where M_t and B_t denote the nominal quantities of outstanding fiat money and government bonds held in the private sector in time t , respectively, and τ_t denote the real value of the lump-sum transfer to each buyer in period t . The government can impose exogenous bank capital requirements to the bankers.

¹¹Tax or transfer is available only for consumption goods.

1.3 Competitive Equilibrium with Lucas tree

In the model a representative banker is assumed to provide a liquidity management service to depositors. Given the aggregate risk the asset holdings can be valuable when the supply of assets is insufficient, but costly when the supply of assets is abundant. A banking arrangement can manage this liquidity provision problem by using a contingent bank capital claim. By providing a proportion of assets to a banker or the other agents when the assets are abundant and providing nothing when the assets are scarce, the liquidity for depositors can be managed efficiently. In this section this contingent banking arrangement is considered to maximize the expected utility of depositors. The optimal banking arrangement can be described as bank capital is held voluntarily even though bank capital is costly to hold.

In the subsections I explore in what circumstance bank capital requirements can improve welfare. Bank capital requirements require additional bank capital holdings for bankers, which restrict the amount of liquidity for depositors in the economy. Thus these capital requirements are not helpful for liquidity provision in general. However, given the aggregate risk, bank capital requirements can be beneficial for smoothing the amount of liquidity across states. When the ex ante asset price reflects the liquidity premium in two states, restricting the liquidity in one state can increase the liquidity in the other state since the asset price is changed by the adjusted liquidity premium in both states.

To focus on these two main ideas in this section I assume that there is no government assets and no reason for liquidity insurance by $\rho = 0$. Under perfect competition bankers suggest a contingent contract to maximize buyers' ex ante expected value. Thus in equilibrium a banker solves the following problem in the *CM* of period t :

$$\underset{d_t, a_t, x_t^h, x_t^l}{Max} -d_t + \pi u(x_t^h) + (1 - \pi)u(x_t^l) \tag{1.1}$$

subject to

$$d_t - \psi_t a_t + \pi \{ \beta(\psi_{t+1} + y^h) a_t - x_t^h \} + (1 - \pi) \{ \beta(\psi_{t+1} + y^l) a_t - x_t^l \} \geq 0 \quad (1.2)$$

$$\beta(\psi_{t+1} + y^h) a_t - x_t^h \geq 0 \quad (1.3)$$

$$\beta(\psi_{t+1} + y^l) a_t - x_t^l \geq 0 \quad (1.4)$$

$$d_t, a_t, x_t^h, x_t^l \geq 0 \quad (1.5)$$

All quantities in (1.1)-(1.5) are expressed in units of the *CM* good in time t . The problem (1.1) subject to (1.2)-(1.5) states that a contingent banking contract (d_t, x_t^h, x_t^l) is chosen in equilibrium to maximize the expected utility of a representative buyer subject to the participation constraint for the banker (1.2) and the incentive constraints for the banker by states (1.3)-(1.4) and non-negativity constraints (1.5). In (1.1)-(1.5) d_t denotes the quantity of goods deposited by the buyer, a_t denotes the demand of the banker for asset holdings, and x_t^i represents the consumption level of the buyer in each state i for $i = h, l$. The quantity on the left side of (1.2) is the net payoff for bankers. In the *CM* of time t the banker receives d_t consumption goods, issues a deposit claim, and invests in the private asset with market prices, $\psi_t a_t$. In the following *CM* the banker pays x_t^h or x_t^l to the holders of the deposit claim by the state h or l . The incentive constraints (1.3)-(1.4) imply that when deposit claims are paid off, the net payoff for the banker is greater than zero, the value that the banker could earn when he or she decides to abscond.

Note that if the limited commitment constraints (1.3) or (1.4) does not bind then bank capital, i.e., asset portfolio minus deposit, is strictly positive in (1.2) because the ex ante profit for bankers must be zero under perfect competition. As well, note that since a state-

contingent contract is considered in the problem, the banker can also choose a non-contingent contract as an optimal choice, if needed.

Government can impose contingent bank capital requirements (δ^h, δ^l) in which a banker must set aside at least $\delta^i \in [0, 1)$ proportion of the asset portfolio by the state i . Then we can have additional bank capital constraints by states,

$$\beta(\psi_{t+1} + y^h)(1 - \delta^h)a_t - x_t^h \geq 0 \quad (1.6)$$

$$\beta(\psi_{t+1} + y^l)(1 - \delta^l)a_t - x_t^l \geq 0 \quad (1.7)$$

where the deposit claim is only pledgeable by $1 - \delta^i$ proportion of the assets in the state i .

Note that for $\delta^i = 0$ the bank capital constraints (1.6)-(1.7) are simply same with the limited commitment constraints (1.3)-(1.4), respectively. For $\delta^i \in (0, 1)$ if the bank capital constraints (1.6)-(1.7) do not bind, the limited commitment constraints (1.3)-(1.4) always do not bind while if the bank capital constraints (1.6)-(1.7) bind then the limited commitment constraints (1.3)-(1.4) are relaxed, respectively. Thus given $\delta^i \in [0, 1)$ an equilibrium can be constructed only with the bank capital constraints (1.6)-(1.7) that replace the limited commitment constraints (1.3)-(1.4) without loss of generality. Notice that bank capital requirements, δ^h and δ^l , are choice variables of government, thus no bank capital requirements with $\delta^h = \delta^l = 0$ can also be chosen at the optimum.

The first step is to solve the problem (1.1) subject to (1.2),(1.5)-(1.7) to characterize equilibrium. The constraint (1.2) must bind, as the objective function is strictly increasing in both x_t^h and x_t^l while (1.2) is strictly decreasing in both x_t^h and x_t^l . Since I will concentrate on the cases either constraint (1.6) or (1.7) binds, let λ^h and λ^l denote the multiplier associated with the incentive constraints (1.6) and (1.7), respectively. Then by plugging (1.2) into (1.1) we have the first-order conditions by a_t, x_t^h, x_t^l ,

$$\psi_t = \pi\beta(\psi_{t+1} + y^h)\{1 + \lambda^h(1 - \delta^h)\} + (1 - \pi)\beta(\psi_{t+1} + y^l)\{1 + \lambda^l(1 - \delta^l)\}, \quad (1.8)$$

$$\pi\{u'(x_t^h) - 1\} = \lambda^h, \quad (1.9)$$

$$(1 - \pi)\{u'(x_t^l) - 1\} = \lambda^l \quad (1.10)$$

which can be reduced to

$$\psi_t = \pi\beta(\psi_{t+1} + y^h)\{(1 - \delta^h)u'(x_t^h) + \delta^h\} + (1 - \pi)\beta(\psi_{t+1} + y^l)\{(1 - \delta^l)u'(x_t^l) + \delta^l\} \quad (1.11)$$

The first-order condition (1.11) states that the net payoff to the banker from acquiring one unit of the asset is zero in equilibrium. In equilibrium a representative bank holds all the assets in its portfolio so that the asset market clear in the *CM* with

$$a_t = 1 \quad (1.12)$$

for $t = 0, 1, 2, \dots$. The market clearing condition (1.12) states that the supply of the private asset is equal to the banker's demand.

Definition 1.1. *Given (π, y^h, y^l) and bank capital requirements (δ^h, δ^l) , a stationary competitive equilibrium consists of quantities (x^h, x^l) and asset price ψ and multipliers (λ^h, λ^l) which satisfy equations (1.6)-(1.10), (1.12).*

Note that there are five variables to be determined in a stationary equilibrium in Definition 1.1 and five equations with the asset market clearing condition are provided. Thus equilibrium allocations are determined by given parameters and bank capital requirements.

From now on I will eliminate t subscripts to restrict the attention to stationary equilibrium allocations.

1.3.1 No Bank Capital Requirements

In this subsection I characterize the equilibrium allocations with no bank capital requirements, $\delta^h = \delta^l = 0$, as a benchmark. Then it will matter for the determination of equilibrium whether the incentive constraints (1.3)-(1.4) bind or not. Thus I will consider each of the three relevant equilibrium cases: Neither constraint binds; the constraint for state l only binds; both constraints bind. Note that there is no equilibrium case in which the constraint for state h only binds since $y^h \geq y^l$ is assumed given $\delta^h = \delta^l = 0$.

Neither constraint binds In this case, since $\lambda^h = \lambda^l = 0$, from (1.8)-(1.10) we have $\psi = \psi^f$ and $x^l = x^h = x^*$ in equilibrium where $\psi_f \equiv \frac{\beta \bar{y}}{1-\beta}$ and x^* satisfies with $u'(x^*) = 1$. The quantity of bank deposits, d , is fixed as x^* in the participation constraint (1.2) since (1.2) holds with equality in equilibrium. The efficient allocation, i.e. the first-best allocation, is attained when both incentive constraints do not bind. That means, given limited commitment, if the supply of the asset is sufficient in an economy, the efficient allocation can be supported. Given $\delta^h = \delta^l = 0$ if the incentive constraint for state l (1.4) does not bind then the incentive constraint for state h (1.3) does not bind as well. Thus it is required to have

$$\beta(\psi^f + y^l) - x^* \geq 0 \quad (1.13)$$

to support the efficient allocation as equilibrium. Equation (1.13), which can be transformed into $\beta y^l + \beta^2 \pi(y^h - y^l) \geq (1 - \beta)x^*$, implies that the efficient allocation is attainable in equilibrium as the expected payoff of the dividend is large enough given the aggregate risk, i.e. $y^h - y^l$. This equilibrium is described as region 1 in Figure 1.3.

Note that holding an asset is not costly in this case since the real return of the asset is

the same as the inverse value of time preference with $\frac{\psi^f + \bar{y}}{\psi^f} = \frac{1}{\beta}$. Thus the bank capital, the asset holdings minus bank deposits, is determined as $\psi^f - x^*$. But it is not costly to hold bank capital in this case.

The constraint for state l only binds In this case since $\lambda^l > \lambda^h = 0$, (4) and (8) can be transformed into

$$\beta(\psi + y^l) - x^l = 0 \quad (1.14)$$

and

$$\psi = \pi\beta(\psi + y^h) + (1 - \pi)\beta(\psi + y^l)u'(x^l), \quad (1.15)$$

respectively. Then the incentive constraint (1.14) and the first-order condition (1.15) solve for ψ and x^l in equilibrium. Since the incentive constraint for state l binds, the consumption level in state l is lower than the optimal level, $x^l < x^*$ and a liquidity premium in the asset price arises so that the asset price is greater than its fundamental value, $\psi > \psi^f$, in equilibrium with $u'(x^l) > 1$. Since the incentive constraint for the state h (1.3) does not bind we have $x^h = x^*$ in equilibrium.¹² The quantity of bank deposits, d , is determined as $\psi - \pi\{\beta(\psi + y^h) - x^*\}$ in the participation (1.2) while the bank capital is $\pi\{\beta(\psi + y^h) - x^*\}$ which is at least positive. Note that both bank deposits and bank capital increase in ψ . This is because when the incentive constraint binds, the asset price rises so that the balance sheet of the banker expands. Additionally, note that even though assets are plentiful in the state h , the asset price, ψ , is greater than its fundamental value, ψ^f , because the asset price, which is determined before the state is realized, also reflects the liquidity premium in the state l .

In this case, since there exists a liquidity premium in the asset price, the real return of the asset is lower than the inverse value of time preference with $\frac{\psi + \bar{y}}{\psi} < \frac{1}{\beta}$. This implies

¹²This case can be generalized with a continuous distribution for dividends. If the variance of dividend distribution is large enough then we will have a measure of h state in which the incentive constraint does not bind.

that holding the asset is costly and so is holding bank capital. However, the bank capital is voluntarily held by the banker in equilibrium since the marginal benefit of holding assets in the state h ex post is lower than the marginal cost of acquiring the asset ex ante. When the state h is realized the marginal benefit of holding extra assets, i.e. the total value of asset portfolio minus the asset used for trade - $\beta(\psi + y^h) - x^*$, is lower than one. This is because the marginal utility of consumption with those extra assets is lower than one with $u'(x^*) = 1$. But the marginal cost of acquiring total asset portfolio is one because the marginal utility of labor supply or consumption good in the *CM* is fixed as one in this quasi-linear model. In order to maximize the depositor's expected value the banker will not let depositors hold these extra assets in the state h ex post. Since the profit of the banker is always zero in equilibrium, it is optimal for the banker to hold bank capital for depositors even though it is costly.¹³ As a consequence bank capital, which is costly to hold, is determined as strictly positive in equilibrium. This implies that bank capital, which is not useful for trade, needs to be held for efficient liquidity management when there exists an aggregate risk in assets and the limited commitment constraint binds.

For this to be an equilibrium, ψ and x^l must satisfy with

$$\beta(\psi + y^h) - x^* \geq 0. \quad (1.16)$$

This implies that given the aggregate risk when the expected payoff of the dividend is small, but the incentive constraint for the state h does not bind, this equilibrium case is feasible. It is described as region 2 in Figure 1.3.

Both constraints bind In this case, since $\lambda^l > 0$, $\lambda^h > 0$, the incentive constraint for the state h (1.3) and the first-order condition (1.8) can be transformed into

¹³Even though the bankers are risk-averse this logic can be applied similarly. The banker will hold extra assets as a bank capital in equilibrium as long as the marginal benefit of holding assets in the state h ex post is the same as the marginal cost of holding assets ex ante.

$$\beta(\psi + y^h) - x^h = 0 \quad (1.17)$$

and

$$\psi = \pi\beta(\psi + y^h)u'(x^h) + (1 - \pi)\beta(\psi + y^l)u'(x^l), \quad (1.18)$$

respectively. Then the incentive constraints (1.14) and (1.17), and the first-order condition (1.18) solve for ψ , x^h , and x^l in equilibrium. Since both incentive constraints bind, the consumption level in the state l is lower than that in the state h , $x^l < x^h$, as long as $y^l < y^h$ holds, and a liquidity premium in the asset price arises so that the asset price is greater than its fundamental value, $\psi > \psi_f$. The quantity of bank deposits, d , is determined as ψ in the participation constraint (1.2) while the bank capital is zero because both incentive constraint bind. The bank capital would not be held by the banker because even in the state h the supply of assets is too scarce so that the marginal benefit of holding the asset is greater than one with $u'(x^h) > 1$. Note that bank deposits increases in ψ as well, but bank capital is fixed as zero in this case because the dividends are too small. When the expected payoff of the dividend is too low given the aggregate risk, this equilibrium case is attainable and it is described as region 3 in Figure 1.3.

In Figure 1.3 region 1 and region 2 are separated by a straight line, i.e. equation (1.13) with equality. The curve between region 2 and 3 is drawn on the points where $x^h = x^*$ just holds with zero bank capital in equilibrium. Thus the incentive constraint for the state h (1.16) holds with equality on this curve. Note that at $y^h = y^l$ region 2 vanishes since the two incentive constraints collapse into one constraint. Thus if there is no aggregate risk then there is no reason to hold costly bank capital for the banker in equilibrium. The dotted line in region 2 indicates the points that provide the same expected payoff of dividends with $\bar{y} = \pi y^h + (1 - \pi)y^l$. This line is located below the borderline between region 1 and 2. It is because given the same level of \bar{y} , the incentive constraint for the state l (1.4) is constrained

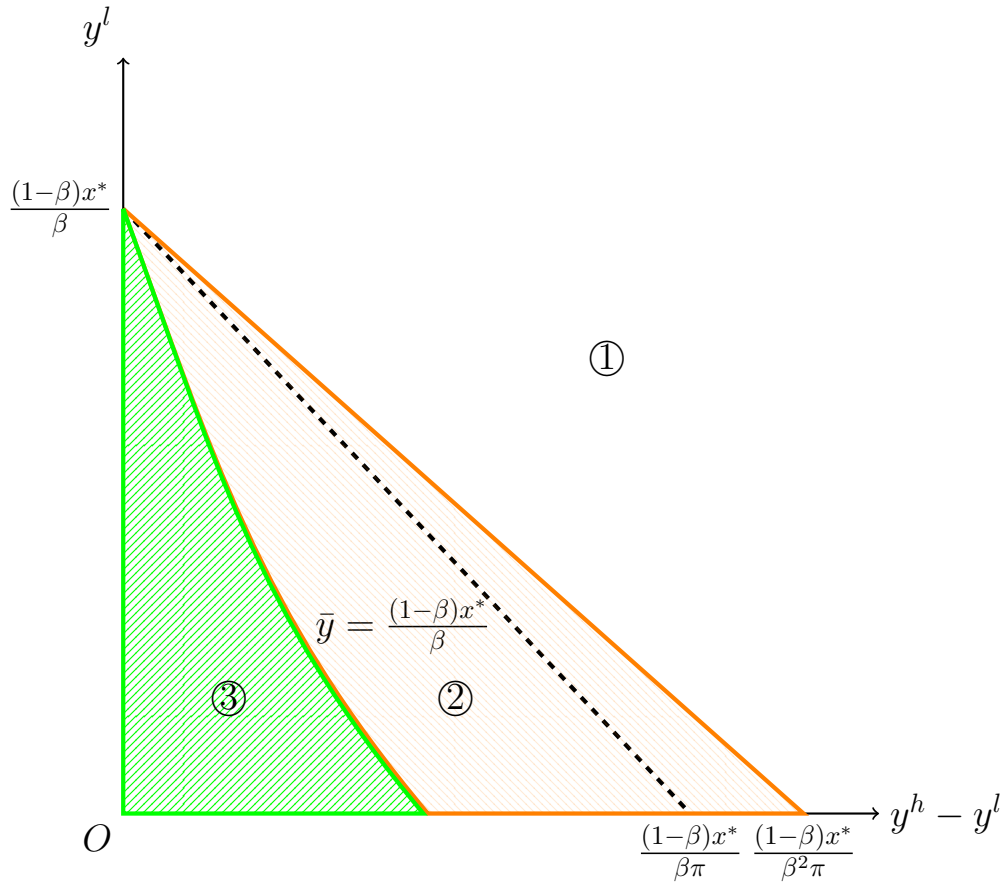


Figure 1.3: Regions with No Bank Capital Requirements

by the lower value of y^l as the aggregate risk increases. Given the same expected payoff of dividend, when the aggregate risk increases the equilibrium allocation moves parallel to the dotted line.

1.3.2 Bank Capital Requirements

In this subsection I consider in what circumstance bank capital requirements can improve welfare. Given the contingent bank capital requirements, we can set either $\delta^h > 0$ or $\delta^l > 0$. If symmetric bank capital requirements with $\delta = \delta^h = \delta^l > 0$ are enforced then the welfare of the equilibrium allocation becomes worse. This is because the symmetric bank capital requirements have the same effect of reducing the supply of assets from one to $1 - \delta$. Thus the consumption levels in both states strictly decrease when the symmetric bank capital requirements are implemented. Then we can consider two asymmetric capital requirements, either $\delta^h > \delta^l = 0$ or $\delta^l > \delta^h = 0$.¹⁴ In the following I focus on the bank capital requirements with $\delta^h > \delta^l = 0$ to verify whether these requirements can be beneficial or not, and to show that the bank capital requirements with $\delta^l > \delta^h = 0$ cannot improve welfare. From now on I assume that $\delta^l = 0$ and replace δ^h with δ . Since the equilibrium allocation is already efficient in region 1, I restrict our attention to the regions 2 and 3.

No Aggregate Risk Let me begin with a special case, in which there is no aggregate risk with $y^h = y^l = \bar{y}$, in order to know the benefit of bank capital requirements. Since the aggregate risk is diversified, the consumption levels in both states are equal as $x^l = x^h \equiv x$ in equilibrium. Then the first-order condition (1.11) can be transformed into

$$\psi = \beta(\psi + \bar{y})u'(x), \quad (1.19)$$

and the incentive constraints (1.3) and (1.4) collapse to one incentive constraint. This

¹⁴In case of $\delta^h > \delta^l > 0$ or $\delta^l > \delta^h > 0$ we can improve the welfare by subtracting δ^l or δ^h in both capital requirements, respectively.

constraint can be written as

$$\beta(\psi + \bar{y})(1 - \delta) - x \geq 0 \quad (1.20)$$

with asset market clearing condition, $a = 1$, in equilibrium. Since we are not interested in the equilibrium case of region 1, suppose that the bank capital constraint (1.20) binds with $\delta = 0$. The bank capital constraint (1.20) states that if $\delta > 0$ then the deposit claim is backed only by $1 - \delta$ proportion of the assets. Given δ , the first-order condition (1.19) and the incentive constraint (1.20) with equality solves for ψ and x in equilibrium. Note that the equilibrium allocation is uniquely determined because ψ is strictly decreasing in x in (1.19) and strictly increasing in x in (1.20).

Lemma 1.2. *If there is no aggregate risk and the incentive constraint binds, the welfare is strictly decreasing in δ .*

Proof. *If the incentive constraint (1.20) binds then $x > 0$ solves for $x(1 - \beta u'(x)) = (1 - \delta)\beta \bar{y}$ in equilibrium. Since $1 - \beta u'(x)$ is strictly increasing in x , x is strictly decreasing in δ*

■

Lemma 1.2 states that given limited commitment, if there is no aggregate risk then bank capital requirements cannot be beneficial. If bank capital requirements are effective in equilibrium, the banker needs to hold more capital than he/she would choose. Thus as long as holding assets is costly, bank capital requirements have a negative effect on the welfare by reducing the proportion of the assets which is useful for trade. Moreover, there is no positive effect of bank capital requirements on the welfare in this case. Note that in this case bank capital requirements are not contingent. They always restrict a fixed δ proportion of the asset. In this respect the reason that bank capital requirements cannot be beneficial in this case can also be explained by the case of the symmetric bank capital requirements.

Aggregate Risk Now consider a general case in which there exists an aggregate risk with $y^h > y^l$. Note that equilibrium in region 2 and equilibrium in region 3 are almost the same except for that there exists a strictly positive bank capital in region 2. So I analyze primarily whether the welfare of the equilibrium in region 2 can be improved by bank capital requirements. The same argument can be applied for the equilibrium in region 3.

Suppose that an equilibrium in region 2 exists with a strictly positive bank capital given $\delta = 0$. Then there exists a threshold $\tilde{\delta} > 0$ at which the bank capital constraint (1.6) starts to bind; At $\delta = \tilde{\delta}$ we still have $\psi = \psi_f$ and $x^h = x^*$ in equilibrium and (1.6) holds with equality. Thus $\tilde{\delta}$ requires to satisfy with

$$\beta(\psi + y^h)(1 - \tilde{\delta}) - x^* = 0 \quad (1.21)$$

where ψ and x^l are the solution of the incentive constraint (1.14) and the first-order condition (1.15). By construction, for $0 \leq \delta \leq \tilde{\delta}$ bank capital requirements are not effective in real allocations because (1.3) does not bind. Thus the equilibrium allocation is the same as one with $\delta = 0$ and only bank capital is decreasing in δ . As a result, bank capital requirements are not beneficial for $0 \leq \delta \leq \tilde{\delta}$ in region 2.

For $\delta > \tilde{\delta}$ bank capital requirements are effective in real allocations since the bank capital constraint (1.6) binds. In this case the first-order condition (1.11) can be written as

$$\psi = \pi\beta(\psi + y^h)\{(1 - \delta)u'(x^h) + \delta\} + (1 - \pi)\beta(\psi + y^l)u'(x^l) \quad (1.22)$$

which can be rearranged to

$$\pi x^h \{u'(x^h) + \frac{\delta}{1 - \delta}\} + (1 - \pi)x^l u'(x^l) = \frac{\pi\beta y^h \{(1 - \delta)u'(x^h) + \delta\} + (1 - \pi)\beta y^l u'(x^l)}{1 - \beta\pi\{(1 - \delta)u'(x^h) + \delta\} - (1 - \pi)\beta u'(x^l)} \quad (1.23)$$

Note that the left-hand side of (1.23) is strictly increasing in x^h and in x^l because $-x \frac{u''(x)}{u'(x)} = \gamma < 1$ while the right-hand side of (1.23) is strictly decreasing in x^h and in x^l .

Thus we can rewrite (1.23) in the form of

$$F(x^h, x^l, \delta) = 0, \quad (1.24)$$

where the function $F(\cdot, \cdot)$ is strictly increasing in both arguments x^h, x^l given δ . Then the first-order condition (1.23) can be describe as the *FOC* curve in Figure 1.4.

Meanwhile, the two binding constraints (1.6) and (1.4) can be written with equality,

$$\beta(\psi + y^h)(1 - \delta) - x^h = 0 \quad (1.25)$$

and

$$\beta(\psi + y^l) - x^l = 0, \quad (1.26)$$

respectively. Note that we have $x^h < x^*$ in equilibrium. Binding incentive constraints (25) and (26) can be reduced to

$$\beta(y^h - y^l) = \frac{x^h}{1 - \delta} - x^l \quad (1.27)$$

where (27) can be described as the *IC* curve in Figure 1.4.

Note that the function $F(\cdot, \cdot)$ is strictly increasing in δ given x^h and x^l , because the left-hand side is strictly decreasing in δ whereas the right-hand side is strictly increasing in δ given x^h and x^l . Thus when δ increases the *FOC* curve shifts towards the origin from FOC_1 to FOC_2 as shown in Figure 1.4. On the other hand, as δ increases the *IC* curve (27) rotates counter-clockwise from IC_1 to IC_2 as shown in Figure 1.4. By rotating the *IC* curve x^h and x^l moves towards the 45 degree line where the consumption levels in both states are equal. Thus the bank capital requirement in the state h can show that the consumption risk can be shared across the states.¹⁵

¹⁵Note that when δ increases, x^h decreases in equilibrium, but we need to confirm that x^l can increase and the welfare can improve.

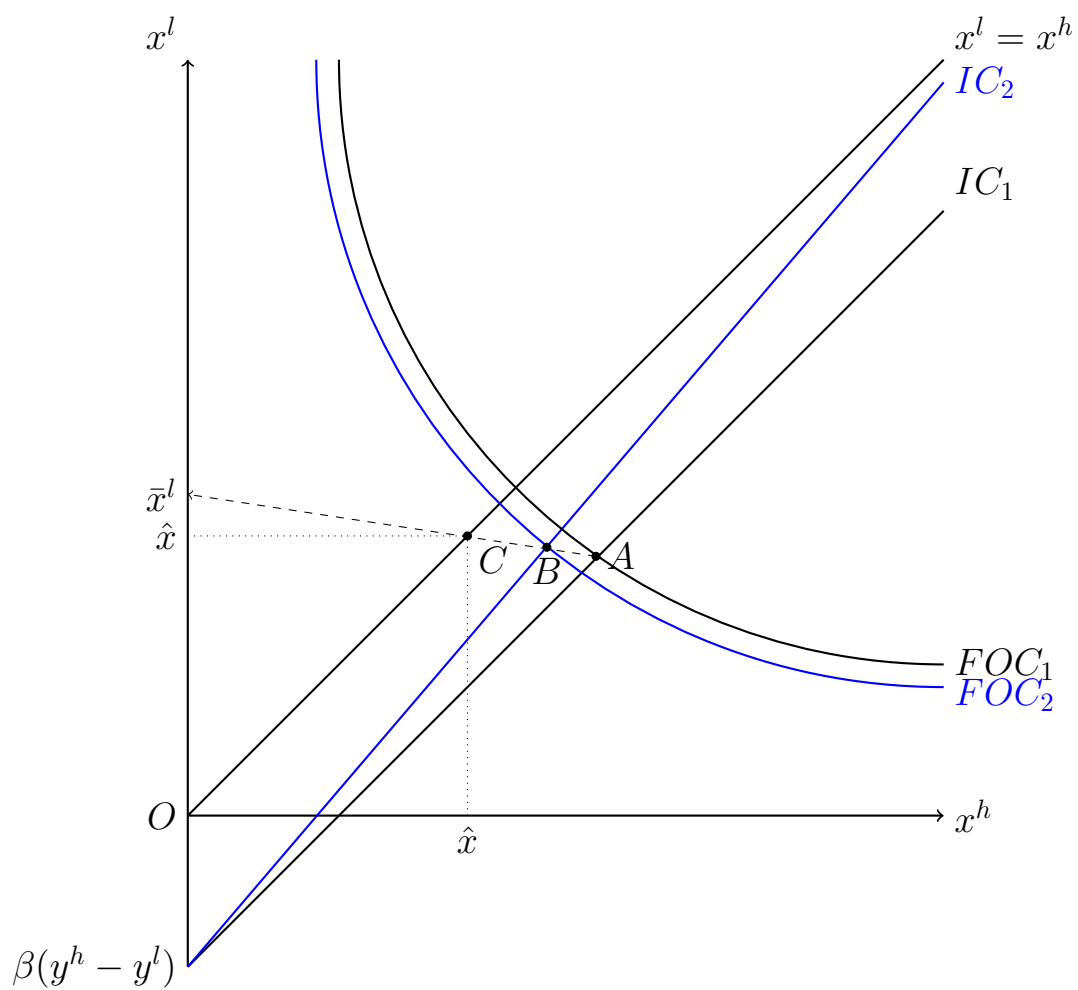


Figure 1.4: Risk-sharing with Bank Capital Requirements

Let me briefly discuss that the bank capital requirements with $\delta^l > \delta^h = 0$ cannot be beneficial. Suppose that the same equilibrium in region 2 exists with a strictly positive bank capital given $\delta^l = \delta^h = 0$ as above. If the bank capital requirements with $\delta^l > \delta^h = 0$ are implemented, then for $\delta^l > 0$ bank capital requirements are effective immediately. But since the incentive constraint for state h (1.16) remains relaxed with $x^h = x^*$ there is no benefit for consumption risk-sharing. It can be also confirmed by the following equilibrium conditions in which the first-order condition and the incentive constraint can be transformed into the bank capital requirements with $\delta^l > \delta^h = 0$ into,

$$\pi x^h u'(x^h) + (1 - \pi) x^l \{u'(x^l) + \frac{\delta^l}{1 - \delta^l}\} = \frac{\pi \beta y^h u'(x^h) + (1 - \pi) \beta y^l \{(1 - \delta^l) u'(x^l) + \delta^l\}}{1 - \pi \beta y^h u'(x^h) - (1 - \pi) \beta y^l \{(1 - \delta^l) u'(x^l) + \delta^l\}} \quad (1.28)$$

and

$$\beta(y^h - y^l) = x^h - \frac{x^l}{1 - \delta^l}, \quad (1.29)$$

respectively. Note that when δ^l increases the first-order condition (1.28) moves into the origin as well, but the incentive constraint (1.29) rotates clockwise. Thus x^h and x^l shifts away from the 45 degree line so that it is worse even in a view of consumption risk-sharing. Hence the bank capital requirements with $\delta^l > \delta^h = 0$ cannot be beneficial.

Now I return to our main subject that the bank capital requirement in the state h with $\delta^h \equiv \delta > \tilde{\delta}$ can be beneficial when the benefit of consumption risk-sharing is greater than the cost of holding enforced bank capital.

Lemma 1.3. *There exists a unique $\hat{\delta} > 0$ where $x^h = x^l$ at $\delta = \hat{\delta}$ in equilibrium.*

Proof. *As $\delta \rightarrow 1$, the equilibrium allocation approaches to $(0, \bar{x}^l)$ where $\bar{x}^l = \min(x^*, x^l)$ such that x^l satisfies with the first-order condition (1.23) and the equilibrium condition (1.27) given $x^h = 0$ and $\delta = 1$ as shown in Figure 1.4. Note that the solution of*

this constrained maximization problem is continuous in δ since u and u' is continuous. Thus as shown in Figure 1.4, by the Intermediate Value theorem, there exists a point that $x^h = x^l$ holds at $\delta = \hat{\delta} \in (\tilde{\delta}, 1)$ in equilibrium. This point is unique because when δ increases x^h strictly decreases ■

Lemma 1.3 is helpful to earn Proposition 1.4. It states that as δ approaches to 1, x^h and x^l must pass the 45 degree line in which the consumption risk is perfectly shared. Thus we can just compare the point A with the point B in the Figure 1.4, because the welfare of the equilibrium located in the upper side of the 45 degree line is lower than the welfare of the point B . Note that the contract at $\delta = \hat{\delta}$ is a non-contingent debt contract because $x^h = x^l$ holds in equilibrium.

Proposition 1.4. *In region 2 the optimal bank capital requirement δ^* exists in $(\tilde{\delta}, \hat{\delta}]$ when agents are sufficiently risk-averse with $\gamma > \gamma^*$.*

Proof. Given $\delta = \hat{\delta}$, $\hat{x} \equiv x^h = x^l$ holds in equilibrium by construction. Then from the first-order condition (1.22) and the binding constraint (1.26), in equilibrium \hat{x} must satisfies with

$$\frac{1}{\beta}\hat{x} - y^l = \hat{\psi} = \hat{x}u'(\hat{x}) + \pi\frac{\hat{\delta}}{1-\hat{\delta}}\hat{x} \quad (1.30)$$

where $\hat{\psi}$ denote the asset price in the equilibrium with $\delta = \hat{\delta}$. Since $\beta(y^h - y^l) = \frac{\hat{\delta}}{1-\hat{\delta}}\hat{x}$ holds from the equilibrium condition (1.27), the equation (1.30) can be transformed into

$$\frac{1}{\beta}\hat{x} - \hat{y} = \hat{x}u'(\hat{x}) = (1-\gamma)u(\hat{x}) \quad (1.31)$$

where $\hat{y} \equiv \pi\beta y^h + (1-\pi\beta)y^l$. The second equality in (1.31) is derived by $-x\frac{u''(x)}{u'(x)} = \gamma$. Thus given \hat{y} and γ , \hat{x} is pinned down from (1.31). Given $\delta = \tilde{\delta}$, since the bank capital constraint (1.6) does not bind, the equilibrium conditions (1.14)-(1.15) and (1.21) can be transformed into

$$\tilde{\psi} = \pi x^* u'(x^*) + \pi x^* \frac{\tilde{\delta}}{1 - \tilde{\delta}} + (1 - \pi) x^l u'(x^l) = (1 - \gamma) \{ \pi u(x^*) + (1 - \pi) u(x^l) \} + \pi x^* \frac{\tilde{\delta}}{1 - \tilde{\delta}} \quad (1.32)$$

where $\tilde{\psi}$ denote the asset price in the equilibrium with $\delta = \tilde{\delta}$, similarly. Let's define $\bar{x} \equiv \pi x^* + (1 - \pi) x^l$ and define \tilde{x} as a certainty equivalent value between x^* and x^l by $u(\tilde{x}) \equiv \pi u(x^*) + (1 - \pi) u(x^l)$. Then there exists a proportion $p \in (0, \pi)$ which satisfies with $\tilde{x} = \frac{p}{1 - \tilde{\delta}} x^* + (1 - p) x^l$ since u is strictly concave. Then (32) can be rewritten as

$$\frac{1}{\beta} \tilde{x} - \tilde{y} = \tilde{\psi} = (1 - \gamma) \left\{ \frac{\pi}{1 - \tilde{\delta}} u(x^*) + (1 - \pi) u(x^l) \right\} = (1 - \gamma) u(\tilde{x}) \quad (1.33)$$

where $\tilde{y} \equiv p y^h + (1 - p) y^l$. Since p is strictly decreasing in γ , \tilde{x} decreases from \bar{x} to x^l as γ increases in $(0, \infty)$. Thus there exists a threshold γ^* where $\tilde{x} = \hat{x}$ holds. Given (π, y^h, y^l) , if $\gamma > \gamma^*$ then $\tilde{x} < \hat{x}$. Finally, the welfare function in the equilibrium with $\delta = \hat{\delta}$ is $\hat{W} = u(\hat{x}) - \hat{x} + \bar{y}$ whereas the welfare function in the equilibrium with $\delta = \tilde{\delta}$ is expressed as $\tilde{W} = \pi \{ u(x^*) - x^* \} + (1 - \pi) \{ u(x^l) - x^l \} + \bar{y}$. Since $\pi \{ u(x^*) - x^* \} + (1 - \pi) \{ u(x^l) - x^l \} \leq \frac{\pi}{1 - \tilde{\delta}} \{ u(x^*) - x^* \} + (1 - \pi) \{ u(x^l) - x^l \} = u(\tilde{x}) - \bar{x} \leq u(\tilde{x}) - \tilde{x} < u(\hat{x}) - \hat{x}$ holds, $\hat{W} > \tilde{W}$ when $\gamma > \gamma^*$. Thus the optimal bank capital requirement δ^* exists in $(\tilde{\delta}, \hat{\delta}]$ because the welfare of the equilibrium allocation is continuous in δ since u is continuous and the solution of the problem is also continuous ■

Corollary 1.5. In region 3 the optimal bank capital requirement δ^* exists in $(0, \hat{\delta}]$ when agents are sufficiently risk-averse with $\gamma > \gamma^*$.

Proof When both incentive constraints (1.6) and (1.4) bind, for any $\delta > 0$ x^h is strictly decreasing in δ because (1.6) already binds. Thus the same proof for Proposition 1 can be applied with $\tilde{\delta} = 0$ ■

Corollary 1.5 states that the same argument can apply for region 3 where the incentive constraint for state h already binds. This is because the consumption risk is also not shared

when both constraints bind.

Proposition 1.4 provides a sufficient condition for beneficial bank capital requirements. Given the aggregate risk and the scarcity of assets, if the risk-aversion of depositors is greater than a threshold, $\gamma > \gamma^*$, no bank capital requirements are no longer the optimal choice of the government. This result implies that bank capital requirements should be considered as a policy tool for consumption risk-sharing. Moreover, this result offers a justification of imposing a procyclical capital requirement, i.e., a counter-cyclical capital buffer in Basel III.¹⁶ A counter-cyclical capital buffer is proposed to mitigate a credit crunch by providing a buffer in recession by accumulating bank capital in a credit boom. However, in this model the procyclical capital requirement can also stabilize the business cycle by adjusting the asset prices without a real transfer.

The intuition behind Proposition 1 is as follows: given the aggregate risk, the supply of assets varies across the states. Given the asset prices a banker can manage the cost of holding assets with bank capital. However, the consumption risk is not shared since the real transfer across the states is hardly achieved. Bank capital requirements can be effective to smooth consumptions by tightening the constraint in the state h which can relax the constraint in the state l . However, this regulation includes a cost of holding an additional bank capital and it is more costly to hold bank capital as assets are scarcer since the liquidity premium rises further. Thus there are three factors which provide a sufficient condition for beneficial bank capital requirements. One is the risk aversion of depositors. Since depositors are risk-averse, they will prefer to pay more for sharing the consumption risk. Secondly, the incentive for risk-sharing becomes stronger as the aggregate risk becomes larger. Finally, the level of asset scarcity is also important because if assets are too scarce, then bank capital becomes too costly to hold for risk-sharing.

Note that the market failure in the problem, which necessitates bank capital requirements,

¹⁶The Basel Committee on Banking Supervision (2010a,b) introduced a new counter-cyclical component which varies from 0 percent to 2.5 percent at regulators' discretion in addition to the minimum total capital requirement.

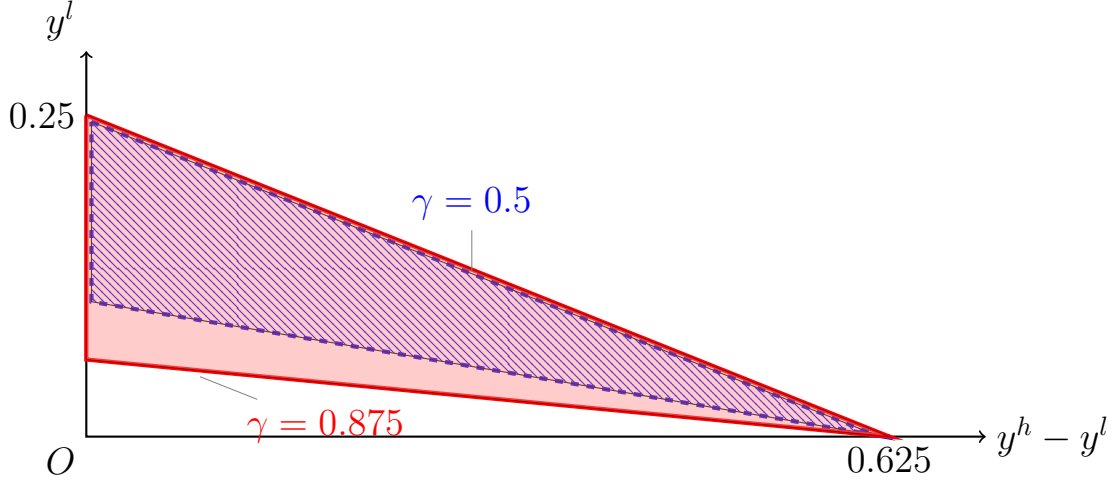


Figure 1.5: Regions of Welfare Improvement by Risk-Aversion

is not related to the deposit contract since a complete contingent contract is considered in the model. It is also not associated with an externality because a representative banker provides ex ante maximized contract to depositors under perfect competition. Thus the incentive of the banker are well aligned with the objective of the society. The market imperfection is caused by limited commitment. Since the transactions in the decentralized market can be supported by the value of the collateral, if there arises a cost for holding the collateral by the scarcity of assets, the first welfare theorem no longer applies.

The proposition 1.4 can be confirmed by a numerical example in Figure 1.5. Given parameter values, the regions, with which the proposition 1.4 is satisfied, are indicated in Figure 1.5. The region does not include the equilibrium cases in which assets are too scarce. Thus the highlighted region widens as the depositors become more risk-averse.

In Figure 1.6 I show numerical examples of different equilibrium allocations: the equilibrium allocation in the panel graphs in the top row is generated for a benchmark in which the optimal bank capital requirement is zero. In this case when δ increases, the liquidity risk is shared because x^h falls and x^l rises. But the welfare strictly decreases since the cost of holding bank capital is greater than the benefit of sharing risk. In the panel graphs in the second row the equilibrium allocation is changed as the total supply of assets increases.

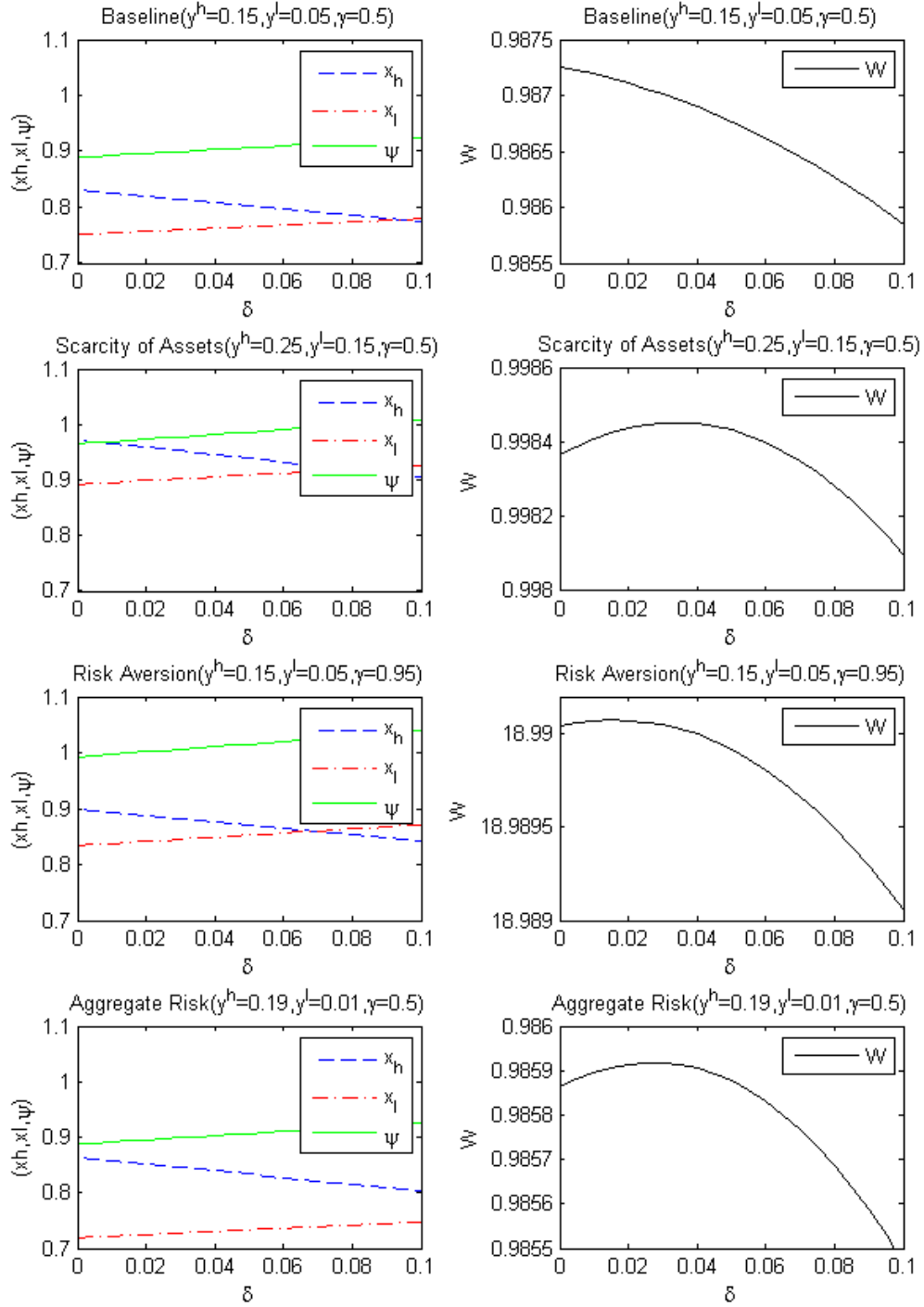


Figure 1.6: Welfare Improvement by Bank Capital Requirements

In this case the welfare can improve as δ increases because the cost of holding bank capital is lowered since the assets are less scarce. Similarly, the welfare can improve by imposing capital requirements when the buyers become more risk-averse as shown in the panel graphs in the third row and when the aggregate risk becomes greater as shown in the panel graphs in the bottom row. Thus these numerical examples confirm that bank capital requirements can be beneficial when assets are not too scarce and the depositors are sufficiently risk-averse and finally aggregate risk is large enough.

1.4 Monetary Equilibrium

In this section I introduce money and government bonds in the model to consider how bank capital requirements can influence the real macroeconomic variables and how they are associated with the implementation of monetary policy. In the previous section it was shown that bank capital requirements can have an impact on the asset price by adjusting the liquidity premium by states. Thus there is a possibility that given a fixed monetary policy, bank capital requirements affect the inflation rate and real interest rates on assets. Conventional monetary policy is limited at the zero-lower-bound since it is irrelevant to exchange currency and government bonds when their prices are same. But if the real allocation can be changed by imposing bank capital requirements then we can influence macroeconomic variables with bank capital requirements even at the zero-lower-bound. This extension also shows what level of nominal interest rates imply that bank capital requirements will be beneficial. Since the currency trades in the non-monitored meetings are now activated, we should consider the effect of bank capital requirements not only on credit arrangements, but also on currency trades.

I assume that the dividend on assets is known after a buyer meets the banker to withdraw currency. This assumption allows us to characterize equilibrium in a simple way because the consumption of the buyer using currency does not depend on the state.

A representative banker solves the following problem in the CM of period t :

$$\underset{d_t, m_t, b_t, a_t, x_{1t}, x_{2t}^h, x_{2t}^l}{Max} - d_t + \rho u(x_1) + (1 - \rho)\{\pi u(x_2^h) + (1 - \pi)u(x_2^l)\} \quad (1.34)$$

subject to participation constraint

$$\begin{aligned} & d_t - m_t - z_t b_t - \psi_t a_t + \left\{ \frac{\beta \phi_{t+1}}{\phi_t} m_t - \rho x_{1t} \right\} + \\ & \pi \left\{ \frac{\beta \phi_{t+1}}{\phi_t} b_t + \beta(\psi_{t+1} + y^h) a_t - (1 - \rho) x_{2t}^h \right\} + \\ & (1 - \pi) \left\{ \frac{\beta \phi_{t+1}}{\phi_t} b_t + \beta(\psi_{t+1} + y^l) a_t - (1 - \rho) x_{2t}^l \right\} \geq 0 \end{aligned} \quad (1.35)$$

and the limited commitment constraint for currency

$$\frac{\beta \phi_{t+1}}{\phi_t} m_t - \rho x_{1t} \geq 0 \quad (1.36)$$

and the limited commitment constraints for deposit claims by states

$$\frac{\beta \phi_{t+1}}{\phi_t} b_t + \beta(\psi_{t+1} + y^h) a_t - (1 - \rho) x_{2t}^h \geq 0 \quad (1.37)$$

$$\frac{\beta \phi_{t+1}}{\phi_t} b_t + \beta(\psi_{t+1} + y^l) a_t - (1 - \rho) x_{2t}^l \geq 0 \quad (1.38)$$

and the bank capital constraints by states

$$\left\{ \frac{\beta \phi_{t+1}}{\phi_t} b_t + \beta(\psi_{t+1} + y^h) a_t \right\} (1 - \delta^h) - (1 - \rho) x_{2t}^h \geq 0 \quad (1.39)$$

$$\left\{ \frac{\beta \phi_{t+1}}{\phi_t} b_t + \beta(\psi_{t+1} + y^l) a_t \right\} (1 - \delta^l) - (1 - \rho) x_{2t}^l \geq 0 \quad (1.40)$$

and non-negative constraints

$$d_t, m_t, b_t, a_t, x_{1t}, x_{2t}^h, x_{2t}^l \geq 0 \quad (1.41)$$

All quantities in (1.34)-(1.41) are expressed in units of the *CM* good in period t . The problem states that a contingent banking contract is chosen in equilibrium to maximize the expected utility of a buyer (1.34) subject to constraints (1.35)-(1.41). In (1.34)-(1.41), m_t and b_t denote the quantities of money and government bonds in terms of the *CM* good in period t held by the banker and x_{jt}^i denote the consumption of type j buyers in the state i in time t *CM* for $j \in \{1, 2\}$ and $i \in \{h, l\}$. Unlike the secured credit arrangement, currency transactions of type 1 buyers are fully backed by real money balances with no risk in (1.36). Note that given the liquidity shock $\rho \in (0, 1)$ this banking contract provides not only liquidity provision service by using deposit claims, but also liquidity insurance for each type.

From now on I focus on a stationary equilibrium where $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ holds for all t , and μ denote the gross inflation rate. Moreover, we will restrict our attention to the cases in which the first-best allocation is infeasible. Then the participation constraint (1.35) and the incentive constraint for currency trade (1.36) and the incentive constraint for the state l (1.38) always bind, while the incentive constraint for the state h (1.37) may bind or not. In order to know whether the same argument for consumption risk-sharing can also be applied in this extended model I focus on the bank capital requirement in the high-return state, δ^h . So from now on let δ replace δ^h as we did in the previous section. As discussed in the previous section given $\delta \in [0, 1)$ equilibrium can be constructed with the bank capital constraint (1.39) instead of the limited commitment constraint (1.37).

Without loss of generality, the first-order conditions by m , b , a can be attained as

$$\frac{\mu}{\beta} = u'(x_1) \quad (1.42)$$

$$z \frac{\mu}{\beta} = \pi \{ (1 - \delta) u'(x_2^h) + \delta \} + (1 - \pi) u'(x_2^l) \quad (1.43)$$

$$\psi = \beta(\psi + y^h) \pi \{ (1 - \delta) u'(x_2^h) + \delta \} + \beta(\psi + y^l) (1 - \pi) u'(x_2^l) \quad (1.44)$$

Incentive constraints (1.36), (1.38)-(1.39) can be rewritten by dropping t subscripts as

$$\frac{\beta}{\mu}m = \rho x_1 \quad (1.45)$$

$$\left\{\frac{\beta}{\mu}b + \beta(\psi + y^h)a\right\}(1 - \delta) \geq (1 - \rho)x_2^h \quad (1.46)$$

$$\frac{\beta}{\mu}b + \beta(\psi + y^l)a = (1 - \rho)x_2^l \quad (1.47)$$

Note that if the bank capital constraint (1.46) does not bind, bank capital is strictly positive in equilibrium. In equilibrium asset markets clear in the *CM* for all t , so that the demands of the representative banker for currency, government bonds, and private assets are equal to the supplies of outstanding government assets and the fixed unit supply of Lucas tree, respectively, as

$$m_t = \phi_t M_t, \quad (1.48)$$

$$b_t = \phi_t B_t, \quad (1.49)$$

$$a_t = 1 \quad (1.50)$$

I assume that the fiscal authority keeps the total value of the outstanding consolidated government debt, V , constant forever. This requires a transfer $\tau_0 = V$ at $t = 0$. Then from the consolidated budget constraints we obtain the real term of lump-sum transfer,

$$\tau_t = \underbrace{\left(1 - \frac{1}{\mu}\right)V}_{\text{seigniorage}} + \underbrace{\frac{1}{\mu}(z - 1)b}_{\text{real interest payment}}, \quad t = 1, 2, 3, \dots, \quad (1.51)$$

where τ_t is required to maintain the constant value of V for the consolidated government debt

in every period. Note that the lump-sum transfer consists of seigniorage from inflation and real interest payment for government bonds. This fixed real value of consolidated government debt assumption allows us to separate monetary policy, specifically open market operations, from fiscal policy. Moreover, by assuming V as being small, we can explore the cases in which the first-best allocation is infeasible.

Since the policy rule of the fiscal authority for $t = 1, 2, 3, \dots$ is fixed, all we need to consider for constructing equilibrium is the government budget constraint for $t = 0$ with $\tau_0 = V$,

$$m + zb = V \tag{1.52}$$

Definition 1.6. *Given (π, y^h, y^l, ρ, V) and the nominal interest rate $\frac{1}{z} - 1$, bank capital requirements (δ^h, δ^l) , a stationary monetary equilibrium consists of quantities (x_1, x_2^h, x_2^l) , asset price ψ , inflation rate μ , and multipliers $(\lambda_1, \lambda_2, \lambda_3)$ which solve equations (1.42)-(1.47), (1.50), (1.52).*

Since we have eight unknown variables with only seven equations in the Definition 1.6, in order to determine an equilibrium one of the two variables, inflation rate μ and the price of government bonds z , is required to be a policy variable. I assume that the central bank chooses the nominal interest rate target, $\frac{1}{z} - 1$, and implements open market operations to achieve its goal in the model. Note that the nominal interest rate of government bonds cannot be negative, i.e. $z \leq 1$, in equilibrium.

1.4.1 No Bank Capital Requirements

In this subsection I describe the equilibrium cases without bank capital requirements as a benchmark and explain how bank capital holdings can be changed by monetary policy. Suppose that there are no bank capital requirements, i.e. $\delta = 0$. I also assume that the supply of public assets is not restricted to support ρ portion of currency transactions,

$$V \geq \rho x^*. \quad (1.53)$$

We have three equilibrium cases which are similar to the regions we have studied in the previous section. If private assets are plentiful in the economy with

$$V + \beta(\psi^f + y^l) \geq x^* \quad (1.54)$$

then the first-best allocation, the Friedman rule equilibrium allocation, is achieved with $x_1 = x_2^l = x_2^h = x^*$, $\mu = \beta$, and $\psi = \psi_f$ which corresponds to the region 1 equilibrium in the previous section. If private assets are scarce, (1.54) violates. Thus in equilibrium both the incentive constraint for currency transactions (1.45) and the incentive constraint for the state l (1.47) bind for sure. But the incentive constraint for the state h (1.46) may bind or not. If (1.46) does not bind then we have the region 2 equilibrium allocations with $x_1 < x^*$, $x_2^l < x_2^h = x^*$, $\mu > \beta$ and $\psi > \psi_f$. If (1.46) binds then we have the region 3 equilibrium allocation with $x_1 < x^*$, $x_2^l < x_2^h < x^*$, $\mu > \beta$ and $\psi > \psi_f$. Note that if (1.46) does not bind, given $\delta = 0$ bank capital is strictly positive in region 2 from the binding participation constraint (1.35). For the same reason bank capital is zero in region 3 when (1.46) binds.

In order to know how these equilibrium cases are associated with monetary policy I characterize equilibrium in region 2 and regions 3 as follows. The first-order conditions, (1.42) and (1.43), can be reduced to

$$zu'(x_1) = \pi u'(x_2^h) + (1 - \pi)u'(x_2^l) \quad (1.55)$$

and binding constraints (1.45) and (1.47), the first-order condition for the private assets (1.44), and the market clearing conditions (1.50) and (1.52) can be transformed into a form of incentive constraint,

$$\begin{aligned}
\rho x_1 u'(x_1) &+ (1-\rho)\pi x_2^h u'(x_2^h) + \frac{(1-\rho)(1-\pi)x_2^l u'(x_2^l)}{\underbrace{\frac{\beta y^h \pi u'(x_2^h) + \beta y^l (1-\pi) u'(x_2^l)}{1 - \beta\{\pi u'(x_2^h) + (1-\pi) u'(x_2^l)\}}}} \\
&= \underbrace{V}_{\text{public asset}} + \underbrace{\phantom{\frac{\beta y^h \pi u'(x_2^h) + \beta y^l (1-\pi) u'(x_2^l)}{1 - \beta\{\pi u'(x_2^h) + (1-\pi) u'(x_2^l)\}}}}_{\text{private asset}}
\end{aligned} \tag{1.56}$$

Note that if (1.46) does not bind then (1.55) and (1.56) can be rewritten by plugging $x_2^h = x^*$ into the equations. If (1.46) binds, from (1.46) and (1.47) we have another equilibrium condition,

$$\beta(y^h - y^l) = (1-\rho)(x_2^h - x_2^l) \tag{1.57}$$

which can be simplified to

$$x_2^h = x_2^l + \alpha \tag{1.58}$$

where α is defined as $\alpha \equiv \frac{\beta(y^h - y^l)}{1-\rho}$. By plugging (1.58) into the first-order condition (1.55) and the incentive constraint (1.56) we can describe the equilibrium allocation (x_1, x_2^l) with two curves as shown in Figure 1.7. In the *FOC* curve (55) x_2^l is strictly increasing in x_1 while in the *IC* curve (1.56) x_2^l is strictly decreasing in x_1 because $-x \frac{u''(x)}{u'(x)} < 1$. Note that x_2^h can be also indicated in the plane since the distance between x_2^h and x_2^l is fixed as α when (1.46) binds. So there is a threshold point $(\tilde{x}_1, \tilde{x}_2)$ on the *IC* curve where the incentive constraint for the state h (1.46) binds at $x_1 > \tilde{x}_1$ because x_2^l is decreasing in x_1 in (1.56). Thus for $x_1 \leq \tilde{x}_1$ the equilibrium allocation is determined with $x_2^h = x^*$ so that this area corresponds to the region 2. For $x_1 > \tilde{x}_1$ the equilibrium allocation is determined with the binding constraint (1.46) with $x_2^h < x^*$ so that this area corresponds to region 3 as shown in Figure 1.6.

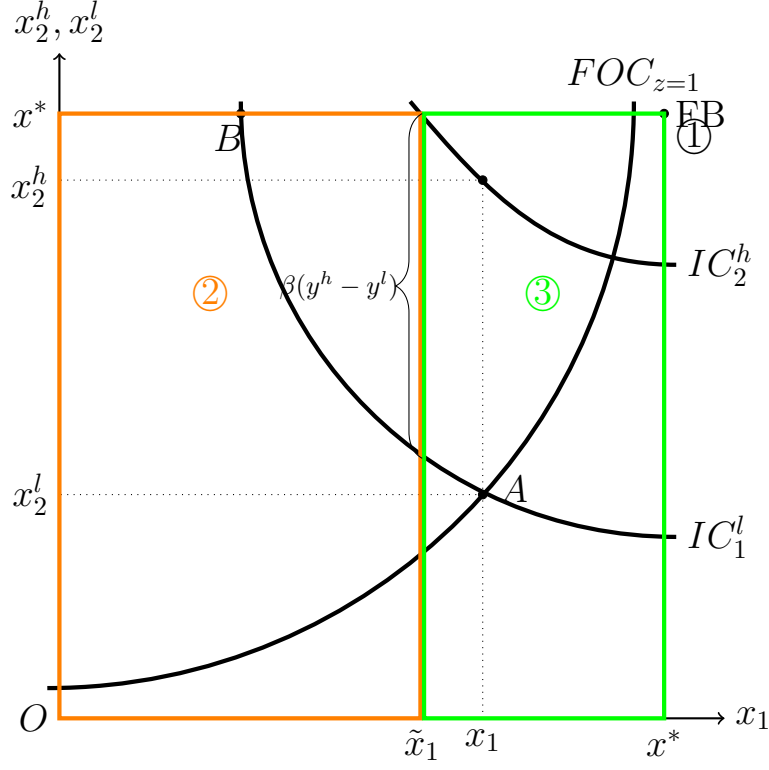


Figure 1.7: Monetary Equilibrium

Given the nominal interest rate target, $\frac{1}{z} - 1$, the equilibrium allocation (x_1, x_2^l) is uniquely determined from (1.55)-(1.56) and x_2^h is passively derived by (1.58). As the nominal interest rate decreases, the FOC curve shifts rightward so that x_1 increases while x_2^h and x_2^l weakly decrease until it arrives at the zero-lower-bound with $z = 1$. This mechanism can be explained as follows: when the central bank injects currency and absorbs government bonds to lower the nominal interest rate, the real return on government bonds falls because the inefficiency in the credit arrangement increases by the less-supplied government bonds, x_2^h and x_2^l , respectively. Meanwhile, the real return on currency, i.e. the inverse of the inflation rate, must increase because people need to hold more real balance of currency, x_1 , in equilibrium.

Given this mechanism of open market operations, when the nominal interest rate falls, bank capital in equilibrium decreases in region 2 and remains zero in region 3. Since both incentive constraints (1.45) and (1.47) bind, bank capital is just derived from the participa-

tion constraint (1.35) in equilibrium as $\pi\{\frac{\beta}{\mu}b + \beta(\psi + y^h) - (1 - \rho)x_2^h\}$ which can be reduced to

$$\pi\{\beta(y^h - y^l) + (1 - \rho)(x_2^l - x_2^h)\} \quad (1.59)$$

by using (1.47). Note that bank capital described in (1.59) strictly declines as x_2^l decreases while x_2^h is fixed as x^* in region 2. Then it becomes zero in region 3 because (1.58) holds in equilibrium. Thus, as the nominal interest rate goes to the zero-lower-bound, the bank capital weakly decreases in equilibrium. This happens because as the nominal interest rate falls, the assets which support credit arrangements become more scarce and so the backed assets become scarce in the state h as well. Also note that if the aggregate risk becomes large, then the area of region 3 shrinks while the area of region 2 expands.

Asset Yields and Liquidity Premium The real interest rate on government bonds in equilibrium can be derived from the first-order condition for government bonds (1.43). It is divided into the fundamental yield and the liquidity premium as

$$r^b = \frac{1 - \beta\{\pi u'(x_2^h) + (1 - \pi)u'(x_2^l)\}}{\beta\{\pi u'(x_2^h) + (1 - \pi)u'(x_2^l)\}} = \underbrace{\left\{\frac{1}{\beta} - 1\right\}}_{\text{fundamental}} - \underbrace{\frac{\{\pi u'(x_2^h) + (1 - \pi)u'(x_2^l)\} - 1}{\beta\{\pi u'(x_2^h) + (1 - \pi)u'(x_2^l)\}}}_{\text{liquidity premium}} \quad (1.60)$$

where the liquidity premium is defined as the difference between the real interest rate on the asset and the fundamental yield from their payoffs. For a strictly positive liquidity premium it is necessary to have an inefficiency of credit trade in at least one state. If the trade is efficient in the *DM*, the consumption is maximized with x^* where $u'(x^*) = 1$. Thus in the term of liquidity premium in (1.60), $u'(x_2^i) > 1$ reflects an inefficiency of credit arrangement in the state i because the incentive constraint for the state i , (1.46) or (1.47), binds. Thus as the assets are more scarce in an economy the liquidity premium on the assets

risks. Note that the liquidity premium on government bonds also reflect the inefficiencies in the states because the price of the government bonds are determined before the next period return is realized.

From the first-order condition for private assets (1.44), the expected real yields on private assets can be derived as

$$r^a = \frac{\bar{y}}{\psi} = \frac{1 - \beta\{\pi u'(x_2^h) + (1 - \pi)u'(x_2^l)\}}{\beta\{\pi u'(x_2^h)\frac{y^h}{\bar{y}} + (1 - \pi)u'(x_2^l)\frac{y^l}{\bar{y}}\}}. \quad (1.61)$$

Note that the rate of return on government bonds can be different from the rate of return on private assets because the denominators are different in (1.60) and (1.61). Since buyers are risk-neutral with respect to the payoff in the *CM*, the fundamental yields from the payoffs of government bonds and private assets are the same. This difference in the rates of return on those assets is generated by the difference in their liquidity premium. Let me define a liquidity-risk premium as a proportional difference in the rates of return on two assets which is described as

$$\frac{r^a - r^b}{r^b} = \frac{\beta\{\pi u'(x_2^h) + (1 - \pi)u'(x_2^l)\}}{\beta\{\pi u'(x_2^h)\frac{y^h}{\bar{y}} + (1 - \pi)u'(x_2^l)\frac{y^l}{\bar{y}}\}}. \quad (1.62)$$

There are two necessary conditions to have a strictly positive liquidity-risk premium. One is an inefficiency of trade in a state i with $u'(x_2^i) > 1$ which generates the liquidity premium on the prices of both assets. If the credit arrangements in both states are efficient then the liquidity-risk premium is zero because the fundamental yields on both assets are the same. The other necessary condition is aggregate risk, i.e. $y^h > y^l$, which provides more weights on the inefficiency of the state h in the denominator than in the numerator in the liquidity-risk premium in (1.62).¹⁷ If there is no aggregate risk with $y^h = y^l$ then the two expected rates of return are the same and the liquidity-risk premium is zero in (1.62). Note that we have $r^b < r^a$ in (1.62) as long as $x_2^l < x_2^h$ holds in equilibrium. This implies that

¹⁷The weight for the expected liquidity premium on private assets is $\{\pi\frac{y^h}{\bar{y}}, (1 - \pi)\frac{y^l}{\bar{y}}\}$ while the weight for government bonds is $\{\pi, (1 - \pi)\}$.

given the inefficiency of credit trade in the state i and the aggregate risk, the rate of return on private assets is greater than the rate of return on government bonds. Also, note that given the inefficiency in both states, if the aggregate risk increases then this liquidity-risk premium increases.

1.4.2 Bank Capital Requirements

In this subsection I consider how bank capital requirements can influence the real interest rates of government bonds and private assets. Moreover, I study when bank capital requirements is welfare-improving given a fixed monetary policy. Suppose that a bank capital requirement, $\delta > 0$, is imposed in the state h only. The equilibrium conditions (1.55)-(1.57) can be modified into

$$zu'(x_1) = \pi\{(1 - \delta)u'(x_2^h) + \delta\} + (1 - \pi)u'(x_2^l), \quad (1.63)$$

$$\begin{aligned} \rho x_1 u'(x_1) + (1 - \rho)\pi x_2^h \{u'(x_2^h) + \frac{\delta}{1 - \delta}\} + (1 - \rho)(1 - \pi)x_2^l u'(x_2^l) \\ = V + \frac{\beta y^h \pi \{(1 - \delta)u'(x_2^h) + \delta\} + \beta y^l (1 - \pi)u'(x_2^l)}{1 - \beta[\pi\{(1 - \delta)u'(x_2^h) + \delta\} + (1 - \pi)u'(x_2^l)]}, \end{aligned} \quad (1.64)$$

$$\beta(y^h - y^l) = (1 - \rho)(\frac{x_2^h}{1 - \delta} - x_2^l), \quad (1.65)$$

respectively. With $\delta > 0$, the threshold point between region 2 and 3, \tilde{x}_1 , moves leftward because the gap between x_2^h and x_2^l in region 3 is reduced by $\delta > 0$ in (1.65). Thus the area of region 3 expands while the area of region 2 shrinks.

The IC curve (1.64) does not change in region 2 because the bank capital constraint (1.46) does not bind when δ increases. But in region 3 as δ increases, the IC curve (1.64) can move upwards as the consumption risk is shared so that x_2^l increases while x_2^h decreases as shown in Figure 7. Notice that the equilibrium conditions (1.64)-(1.65) are similar to (1.22) and (1.26), respectively, except for the terms $V - \rho x_1 u'(x_1)$ in (1.64) and $1 - \rho$ in

(1.65). This implies that given x_1 , two curves for x_2^h and x_2^l can become closer in region 3 as δ increases while the total feasible quantities of (x_2^h, x_2^l) decrease by the $\frac{\delta}{1-\delta}$ term in (1.64) by Proposition 1.

The *FOC* curve (1.63) rotates as δ increases. Since the equilibrium condition (1.65) can be rewritten as $x_2^h = (1 - \delta)(x_2^l + \alpha)$ where $\alpha = \frac{\beta(y^h - y^l)}{1 - \rho}$, the right-side of the first-order condition (1.63) can be transformed to $(1 - \delta)u'((1 - \delta)(x_2^l + \alpha)) + \delta$. Then given x_1 when δ increases there is a tradeoff between an intensive margin effect and an extensive margin effect. The intensive margin effect implies that the liquidity premium rises by the inefficiency of trade by the reduced pledgeability whereas the extensive margin effect implies that the liquidity premium falls because the inefficiency is only applied for the pledgeable part of the assets.

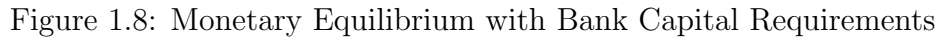
Lemma 1.7. *Given γ and x_1 the FOC curve rotates since if δ increases at $\delta = \tilde{\delta}$, for $x_2^l < \bar{x}_2(\tilde{\delta})$, x_2^l decreases, i.e. $\frac{\partial x_2^l}{\partial \delta} < 0$ while for $x_2^l > \bar{x}_2(\delta)$, x_2^l increases, i.e. $\frac{\partial x_2^l}{\partial \delta} > 0$.*

Proof. *Given the left side of the first-order condition (1.63), $zu'(x_1)$, fixed, by the implicit function theorem at $\delta = \tilde{\delta}$ we have*

$$\frac{\partial x_2^l}{\partial \delta} = -\frac{\pi\{1 - (1 - \gamma)u'((1 - \tilde{\delta})(x_2^l + \alpha))\}}{\pi(1 - \tilde{\delta})^2(u''((1 - \tilde{\delta})(x_2^l + \alpha)) + (1 - \pi)u''(x_2^l))} \begin{matrix} \geq \\ \leq \end{matrix} 0. \quad (1.66)$$

Since the denominator of (1.66) is strictly negative, given γ if $x_2^l < \bar{x}_2(\tilde{\delta})$ where $\bar{x}_2(\tilde{\delta})$ satisfies with $1 = (1 - \gamma)u'((1 - \tilde{\delta})(\bar{x}_2(\tilde{\delta}) + \alpha))$, then we have $\frac{\partial x_2^l}{\partial \delta} < 0$ and otherwise $\frac{\partial x_2^l}{\partial \delta} \geq 0$. Thus the FOC curve rotates counter-clockwise with the center of $\bar{x}_2(\tilde{\delta})$ ■ Since currency is available to use in both currency trade and credit arrangements, given the nominal interest rate, rates of return on currency and government bonds must be equal in the first-order condition. Lemma 1.7 implies that when δ increases, the real interest rate on government bonds is adjusted so that the real interest rate on currency, i.e. the inverse of the inflation rate, must also change. Note that from (1.66) $\bar{x}_2(\delta)$ is strictly decreasing in γ . Thus when γ is sufficiently high, the FOC curve tends to shift leftward further. Also note that $\bar{x}_2(\delta)$ is strictly increasing in δ

This result has an implication on the monetary policy. Since the *FOC* curve can shift leftward by imposing bank capital requirements, the feasible set of equilibrium allocation by choosing monetary policy can be shrunk. For example, the initial allocation x_1 and the inflation rate μ , which is feasible with $z = 1$ and $\delta = 0$, can be no longer feasible with $z = 1$ and $\delta > 0$ if the *FOC* curve moves leftward.



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FOC curve (1.63) rotates. If x_1 is maintained as before then by the first-order condition for currency trade (1.42) the inflation rate does not change. Since the nominal interest rate is fixed, the real interest rate on government bonds does not change either from (1.43) and (1.60). However, the real interest rate on private assets decreases because the liquidity-risk premium with $\delta > 0$ in (1.67) decreases. In (1.67) the numerator does not change since z and x_1 are maintained in the first-order condition (1.63). But the denominator increases because when δ is raised, x_2^l increases in equilibrium so that $u'(x_2^l)$ decreases whereas $(1 - \delta)u'(x_2^h) + \delta$ increases. As a result r^b remains at its original level while r^a adjusts downward.

$$\frac{r^a - r^b}{r^b} = \frac{\beta[\pi\{(1 - \delta)u'(x_2^h) + \delta\} + (1 - \pi)u'(x_2^l)]}{\beta[\pi\{(1 - \delta)u'(x_2^h) + \delta\}\frac{y^h}{y} + (1 - \pi)u'(x_2^l)\frac{y^l}{y}]} \quad (1.67)$$

Similarly, when δ increases if x_1 is determined at the lower level of the original x_1 , the inflation rate goes up so that the real return of government bonds decreases. Then the real return on private assets also decreases because both the liquidity-risk premium and the real interest rate on government bonds decreases. Finally, if x_1 is determined at the higher level of the original x_1 , the inflation rate falls and the real interest rate on government bonds increases. Then the direction of the real interest rate on private assets is ambiguous because the liquidity-risk premium decreases while the real interest rate on government bonds increases.

This result implies that given a fixed monetary policy, bank capital requirements can adjust real interest rates on government bonds and private assets in equilibrium. This means that bank capital requirements can also be effective at the zero-lower-bound where monetary policy is limited to lower the real interest rates further. In this respect bank capital requirements can also be considered as an unconventional policy option at the zero-lower-bound.

Welfare-improving Bank Capital Requirements In this subsection I analyze when bank capital requirements will be beneficial given a fixed monetary policy. As shown in

the previous section, bank capital requirements are beneficial for sharing consumption risk in credit arrangements. However, in this extended model there are currency transactions as well. Thus the welfare improvement of bank capital requirements also depends on how capital requirements influence currency exchanges in the *DM*. Suppose that when δ increases, the *FOC* curve shifts leftward more than the *IC* curve shifts rightward. Then x_1 decreases and the inflation rate goes up in equilibrium. This implies that the inefficiency in the currency trade increases by imposing bank capital requirements. Thus, although the credit arrangements can improve as the consumption risk is shared, the currency trade can be worse off.

Lemma 1.8. *Given $\delta \geq 0$, when the allocation x_1 increases and x_2^l decreases by moving along the *IC* curve, the welfare improves.*

Proof. *Given δ if the nominal interest rate decreases then the equilibrium allocation moves along the *IC* curve. Thus I consider that given δ and the nominal interest rate, $\frac{1}{\hat{z}} - 1$, the allocation is welfare-improving as \hat{z} increases. If we add up the expected utilities across agents in a stationary equilibrium, the welfare measure in the extended model is described as*

$$W = \rho\{u(x_1) - x_1\} + (1 - \rho)\pi\{u(x_2^h) - x_2^h\} + (1 - \rho)(1 - \pi)\{u(x_2^l) - x_2^l\} + \bar{y} \quad (1.68)$$

that represents the sum of surpluses from trade in the DM. Suppose that there exists a unique equilibrium in region 3 given the nominal interest rate, $\frac{1}{\hat{z}} - 1$. In region 3 since the incentive constraint for state h also binds, from (1.63) and (1.65) we have the modified first-order condition,

$$\hat{z}u'(x_1) = \pi\{(1 - \delta)u'((1 - \delta)(x_2^l + \alpha)) + \delta\} + (1 - \pi)u'(x_2^l), \quad (1.69)$$

and from (1.64) and (1.65) the modified equilibrium condition,

$$V + K(x_2^l) = \rho x_1 u'(x_1) + (1 - \rho)\pi x_2^h \{u'(x_2^h) + \frac{\delta}{1-\delta}\} + (1 - \rho)(1 - \pi)x_2^l u'(x_2^l) \quad (1.70)$$

where $K(x_2^l) = \frac{\beta y^h \pi \{ (1-\delta)u'((1-\delta)(x_2^l + \alpha)) + \delta \} + \beta y^l (1-\pi)u'(x_2^l)}{1 - \beta \{ \pi \{ (1-\delta)u'((1-\delta)(x_2^l + \alpha)) + \delta \} + (1-\pi)u'(x_2^l) \}}$. In the (x_1, x_2^l) plane the slope of welfare function (1.68) with $x_2^h = (1 - \delta)(x_2^l + \alpha)$ at $z = \hat{z}$ is

$$\frac{\partial x_2^l}{\partial x_1} = - \frac{\rho \{u'(x_1) - 1\}}{(1 - \rho)[\pi \{ (1-\delta)u'((1-\delta)(x_2^l + \alpha)) - (1-\delta) \} + (1 - \pi)(u'(x_2^l) - 1)]} = - \frac{\rho}{(1 - \rho)\hat{z} - \frac{(1-\hat{z})(1-\rho)}{u'(x_1)-1}} \quad (1.71)$$

while the slope of the equilibrium condition (1.70) at $z = \hat{z}$ is

$$\frac{\partial x_2^l}{\partial x_1} = - \frac{\rho(1-\gamma)u'(x_1)}{(1-\rho)(1-\gamma)[\pi \{ (1-\delta)u'((1-\delta)(x_2^l + \alpha)) + \delta \} + (1-\pi)u'(x_2^l)] + (1-\rho)\pi\gamma\delta - K'(x_2^l)} = - \frac{\rho}{(1-\rho)\hat{z} + \frac{(1-\rho)\pi\gamma\delta - K'(x_2^l)}{u'(x_1)-1}} \quad (1.72)$$

Then the slope of welfare function is steeper than the slope of the equilibrium condition (1.70) since $K'(x_2^l) < 0$. Thus given bank capital requirements, the welfare improves when the allocation x_1 increases and x_2^l decreases along the IC curve ■

Lemma 1.8 shows that the welfare improves as the allocation x_1 increases and x_2^l decreases along the IC curve. Thus we can divide the effect of bank capital requirements on the equilibrium allocation into two different factors. One is the risk-sharing effect by which the allocation in the point A moves to the allocation in the point B in Figure 1.8. The other is the illiquidity effect by which the allocation in the point B moves to the allocation in the point C in Figure 1.8. Thus by the illiquidity effect, a higher quantity of goods is traded in credit arrangements while a lower quantity of goods is traded in currency transactions. Note that the risk-sharing effect improves welfare, but the illiquidity effect on the welfare is ambiguous because the direction of the new equilibrium allocation depends on the degree of the shifts in both FOC and IC curves.

Proposition 1.9. *If agents are sufficiently risk-averse with $\gamma > \gamma^*$ and the equilibrium allocation x_1 increases by δ in a neighborhood of $\delta = 0$, then the welfare improves by imposing $\delta > 0$.*

Proof *Given the risk-aversion of agents $\gamma > \gamma^*$ and the nominal interest rate \hat{z} , suppose that the equilibrium allocation (\hat{x}_1, \hat{x}_2^l) exists in region 3. Define an equilibrium allocation as $(\tilde{x}_1, \tilde{x}_2^l)$ which is determined with $\delta = \tilde{\delta} > 0$. Then the movement from (\hat{x}_1, \hat{x}_2^l) to $(\tilde{x}_1, \tilde{x}_2^l)$ is divided into two parts. One is the movement of x_2^l along the vertical line at $x_1 = \hat{x}_1$. The other is the movement that x_1 increases and x_2^l decreases along the changed IC curve. The welfare improves from the first movement by Proposition 1 and also improves from the second movement by Lemma 4 ■*

The Proposition 1.9 implies that if the sufficient condition for Proposition 1.4, $\gamma > \gamma^*$, is satisfied and currency transactions increase by the shift of the *FOC* curve then bank capital requirements can improve welfare as a sufficient condition. Note that as long as the benefit of sharing consumption risk is greater than the cost of holding additional capital and the additional inefficiency in currency exchange, bank capital requirements are beneficial for society. Thus even though x_1 decreases by imposing bank capital requirements, the welfare can improve if the risk-sharing effect dominates the illiquidity effect.

Notice that this illiquidity effect in which the allocation moves along the *IC* curve is similar to the effect of open market operations because the quantities of currency trade and credit arrangements are adjusted. However, this illiquidity effect is different because it is generated by affecting prices through capital requirements instead of changing the supply of liquid and illiquid assets through open market operations. This result implies that bank capital requirements can also function as a monetary policy tool without exchanging liquid and illiquid assets.

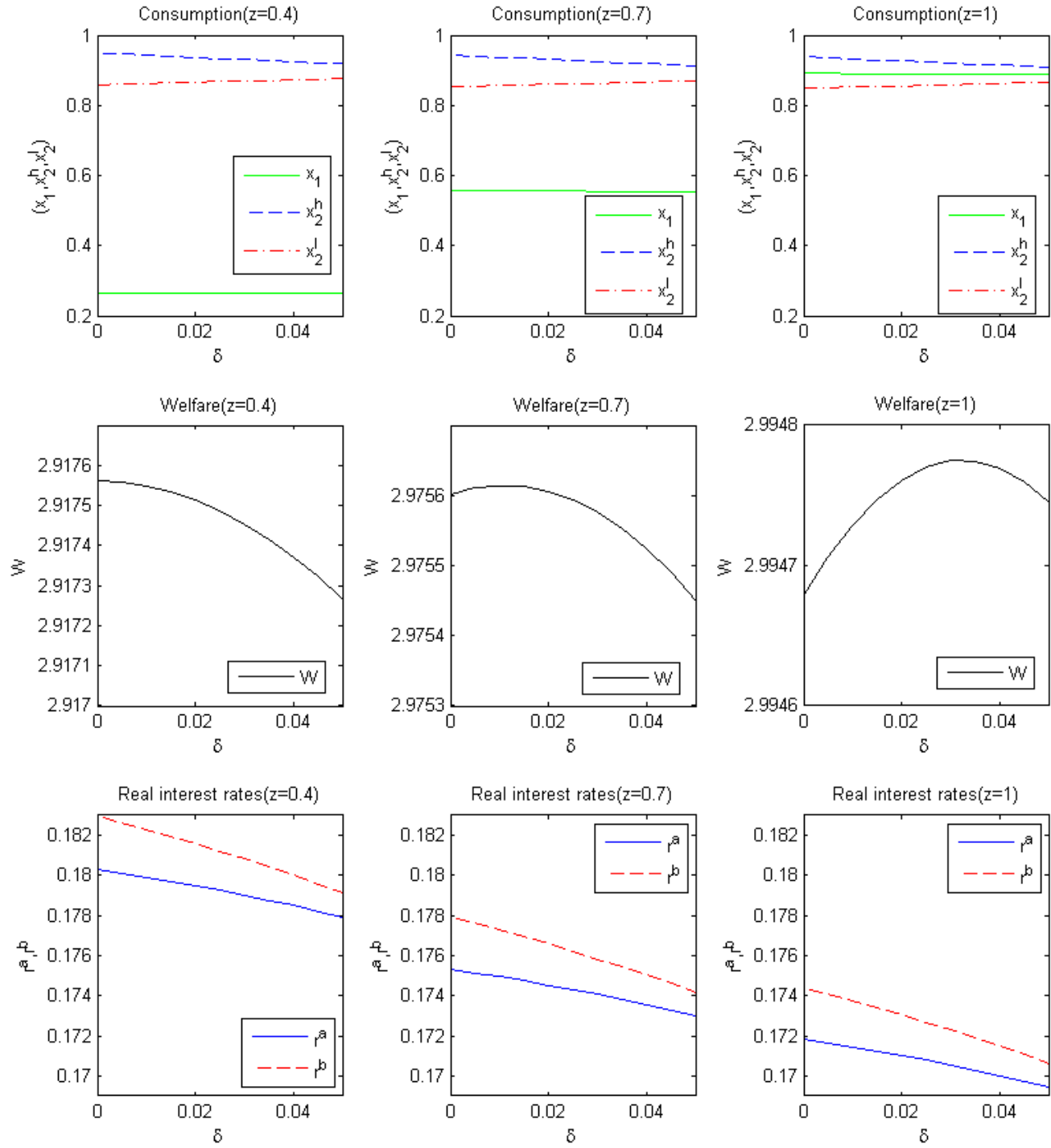


Figure 1.9: Welfare Improvement of Bank Capital Requirements in Monetary Equilibrium

These results are also shown in Figure 1.9 with numerical examples. In the right-side panel graphs given a zero nominal interest rate where the conventional monetary policy is restricted, imposing capital requirements can reduce real interest rates on assets further as discussed. Note that the difference between real returns on government bonds and on private assets is decreasing in δ since consumption risk is shared in credit arrangements. In this case the welfare improves as δ increases although x_1 decreases. This is because the benefit of sharing risk in credit arrangements is greater than the cost of holding bank capital and the inefficiency in currency exchange. With the middle and the left panel graphs, it is also found that bank capital requirements will be beneficial as the nominal interest rate approaches to zero. As shown in the upper panel graphs currency exchange, x_1 , does not change by imposing capital requirements in each case. However, the cost of holding additional bank capital can be different by cases. Since total assets become less scarce when the nominal interest rate approaches zero by Lemma 1.8, the cost of holding bank capital is decreasing in z .¹⁸ Thus in this numerical example bank capital requirements will be beneficial as the nominal interest rate approaches zero because the cost of holding bank capital is decreasing in z .

1.5 Conclusion

I construct a banking model in which a contingent deposit contract is chosen to provide liquidity efficiently given aggregate risk. With limited commitment, deposit claims are backed by bank assets, so that a liquidity premium on assets can arise when the supply of assets is insufficient for efficient exchange in at least one state. In this case it is costly to hold bank capital, but by holding bank capital the banker can manage liquidity for the depositors in an efficient way. A pro-cyclical bank capital requirement, which forces bankers to hold

¹⁸In monetary equilibrium the cost of holding assets does not depend only on the level of the real interest rate on private assets because bankers also hold currency for their asset portfolio. In this case the cost of holding currency is lower as the nominal interest rate goes to zero since the level of currency exchange, x_1 , is increasing in z .

additional bank capital in the high-return state, can improve welfare by smoothing consumption. Although it is costly to hold additional bank capital, reducing the pledgeability of the assets in the high-return state can relax the incentive constraint in the low-return state by affecting the asset price. In the extended model with money and government bonds, the relationship between bank capital requirements and monetary policy is studied. Since bank capital requirements adjust the pledgeability of the assets, bank capital requirements can influence macroeconomic variables such as the inflation rate and real interest rates on the assets. If agents are risk-averse enough, bank capital requirements will be beneficial as the nominal interest rate approaches zero.

This paper takes steps to understand the role of bank capital for efficient liquidity provision. It also sheds light on the rationale for bank capital requirements as a macro-prudential policy that accommodates risk-sharing by affecting the pledgeability of assets. This implication is consistent with recent empirical studies in which macro-prudential policy tools are shown as effective in stabilizing credit-cycles. Lim et al. (2011) find that several macro-prudential tools, such as the Loan-to-Value ratio cap, dynamic provisioning, and the counter-cyclical buffer, can reduce the pro-cyclicality of credit growth by using the 2011 IMF survey data. Akinci and Olmstead-Rumsey (2015) develop a new index of macro-prudential policies in 57 countries and show that macro-prudential policy variables exert a negative effect on bank credit growth with a dynamic panel data model. However, this result cannot provide an answer for welfare issues because the cost of externality is given exogenously in the previous theoretical models. This paper can contribute to this growing literature by providing a relevant justification for welfare improvement with a theoretical model in which the cost of holding capital is endogenously chosen.

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Chapter 2

Scarcity of Assets, Private Information, and the Liquidity Trap

2.1 Introduction

The liquidity trap, in which monetary policy is no longer effective, has been a subject of interest to both monetary theorists and central bankers since the Great Depression of the 1930s. In particular, excess reserves are associated with the liquidity trap because exchanging reserves with government bonds is ineffective to real allocations. One conventional explanation for excess reserves is based on lack of good loan opportunities. For example, in recessions commercial banks would hold excess reserves voluntarily since the expected return of projects is lower than the rate of return in reserves. The other widespread view of excess reserves is that banks accumulate reserves to offset the aggregate liquidity shock such as large withdrawals in banking panics. In this paper I develop a new theory of the liquidity trap in which excess reserves are useful for providing liquidity insurance efficiently by revealing private information.

Providing liquidity insurance is one of the primary functions of banks. When people are exposed to idiosyncratic liquidity risk, separating the types by liquidity needs *ex post* is

beneficial for efficient liquidity distribution. For example, a banking contract can provide liquid assets to impatient agents with low returns while promising high returns to patient agents with illiquid assets. If the returns for the patient types are sufficiently high, private information on the types does not matter because the patient types would prefer to earn high returns. However, if the rate of return for patient types is sufficiently low then the patient types have an incentive to mimic impatient types so that separating the types is difficult under private information. In this case in order to separate the types, banks are required to hold sufficient assets, either liquid or illiquid, for patient types. In this respect excess reserves can exist in the balance sheet of banks to distribute liquidity efficiently by types. In this case if the real value of total assets is fixed, adjusting the proportion of liquid and illiquid assets is no longer effective in real allocations.

In order to explore this issue I construct an asset-exchange model in which two different liquid assets can be used for separating the types. This micro-founded model is useful for incorporating informational frictions such as lack of memory and limited commitment, and it is also highly tractable with a banking contract. The basic structure of the model builds on Lagos and Wright (2005), specifically Rocheteau and Wright (2005), where *ex ante* heterogeneous agents trade in decentralized meetings and rebalance their portfolios in a centralized market. The form of banking contract comes from Williamson (2012) where a banking contract provides liquid assets for asset exchange and illiquid assets for credit arrangements. There are fixed supplies of both private and government assets that are insufficient to support the optimal level of consumption in both exchanges. Given the supply of assets, a banking contract is considered to maximize the *ex ante* expected value of depositors under perfect competition.

One important assumption is lack of record-keeping technology. This anonymity assumption makes recognizable assets essential for decentralized trade. Simultaneously, when agents are subject to individual liquidity risk, this imperfect memory inhibits banks in revealing

the types.¹ Thus a banking contract with truth-telling constraints is considered to provide liquidity insurance efficiently under private information.

This paper provides two key findings. First, when the total supply of assets is insufficient to separate the types, a liquidity premium could arise in the price of illiquid assets even though illiquid assets are not useful for trade directly. It is because illiquid assets are useful for revealing the private type information. After the individual liquidity risk is realized, the optimal liquidity distribution of banks is to provide liquid assets to the impatient types. However, the patient types always have an incentive to mimic the impatient types. Thus banks are required to hold liquid or illiquid assets to inhibit the impatient types from withdrawing liquid assets. Thus when the supply of assets is insufficient to separate the types, the liquidity premium can arise in either liquid or illiquid asset prices.

Secondly, when the total supply of assets is insufficient to separate the types, a proportion of liquid assets, i.e. excess reserves, should be held in balance sheets of banks for the patient types to reveal their types. In this case although the government injects money in the markets by absorbing government bonds, the currency trade does not increase since the injected money would be just held as reserves to reveal the types. This liquidity trap equilibrium can exist when the truth-telling constraint binds to reveal the private information. Thus it could exist away from the zero-lower-bound in which the conventional liquidity trap equilibrium exists: a liquidity trap can exist when the rates of return in currency and government bonds are the same at the zero nominal interest rate.

The first finding is related to the literature about the liquidity premium on asset prices. Geromichalos et. al. (2007) and Lagos and Rocheteau (2008) show that the asset prices can have a liquidity premium when the assets are useful for exchange. This paper is different from these papers because illiquid assets are not useful for trade although there exists a liquidity premium on those assets when the supply of assets is insufficient. This usefulness of illiquid assets is also different from the reasons why the illiquid bonds are beneficial in

¹If record-keeping is available, credit or (proportional) tax scheme can replicate the optimal equilibrium allocation even under private information.

Kocherlakota (2003) and Shi (2008). Kocherlakota (2003) shows that illiquid bonds can relax the cash-in-advance constraint as agents can trade assets after observing idiosyncratic shock. Shi (2008) shows that the welfare improves when government bonds are legally restricted for one type of trade. In those papers illiquid assets are useful because they are less liquid than liquid assets, but in this paper both liquid and illiquid assets can be useful to separate the types.

The second result is related to the literature on the implementation of monetary policy. Wallace (1981) studies the effectiveness of monetary policy, in particular open market operations, by applying the Modigliani-Miller theorem to the government liability structure. In this paper excess reserves can exist in equilibrium when the truth-telling constraint binds so that monetary policy can be ineffective. These excess reserves are different from the ones in the liquidity trap as shown in Williamson (2012) because excess reserves are uniquely determined to reveal the types in this paper.

This paper is related to the literature on liquidity insurance. In their pioneering paper Diamond and Dybvig (1983) show that bank runs can exist as an equilibrium outcome when liquidity shocks are private information. Allen and Gale (1998) point out that the rates of return on long-term projects are critical to revealing the type information in the Diamond-Dybvig type model. In this paper private information is emphasized as a main friction. Moreover, there is no coordination failure of patient agents as shown in Diamond and Dybvig (1983) and no default by aggregate risk as in Allen and Gale (1998).

This paper builds on the previous banking models with an explicit asset trade. Freeman (1988) and Champ, Smith, and Williamson (1996) study banking and liquidity insurance with overlapping generation models. Recently, Bencivenga and Camera (2011) have used uncertainty in trading opportunity in the Lagos and Wright (2005) framework to introduce an insurance banking, but a standard debt contract is considered without individual incentive constraints as shown in their paper. Williamson (2012) constructs a Diamond-Dybvig type bank in the Rocheteau and Wright (2005) framework and shows that a liquidity trap can

exist away from the Friedman rule.

The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 offers a simple model with two different liquidity assets and compares the equilibrium allocations under perfect information and private information. Section 4 extends the model with government liabilities and monetary policy and analyzes in what circumstance monetary policy can be ineffective. Section 5 discusses the result to address implications, and Section 6 concludes.

2.2 Model

The basic structure of the model is based on Rocheteau and Wright (2005). Time $t = 0, 1, 2, \dots$ is discrete in infinite horizon and each period is divided into two sub-periods - the Centralized Meeting (*CM*) followed by the Decentralized Meeting (*DM*). The population consists of two types of *ex ante* heterogeneous agents, buyers and sellers, who are infinitely lived with $[0, 1]$ continuum of each type. An individual buyer has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + \theta_t^i u(x_t)]$$

where $H_t \in \mathbb{R}$ is labor supply in the *CM*, $x_t \in \mathbb{R}_+$ is consumption in the *DM*, and $0 < \beta < 1$. Assume that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u'(0) = \infty$, $u(0) = 0$, and $-x \frac{u''(x)}{u'(x)} < 1$ for all $x > 0$.² One variation is that each buyer is exposed to an idiosyncratic liquidity shock, θ_t^i , which follows independent and identical distribution with two realizations $\{1, \theta\}$ where $\theta \in [0, 1)$ by types $i = 1, 2$. With probability $\rho \in (0, 1)$ a buyer can be type 1 with $\theta_t^1 = 1$ and otherwise a buyer is type 2 with $\theta_t^2 = \theta$. There can be *ex post* heterogeneity in marginal utility across the buyers.³ Each seller has

²In the model asset demand is strictly increasing in rates of return when the coefficient of relative risk aversion is less than one. It guarantees to have at least one equilibrium exists.

³This is different with Shi (2008) where illiquid bonds are beneficial because it can be used for higher marginal utility trade. It is also different with He, Huang and Wright (2008) where theft on money trade can lead higher marginal utility in deposit claims trade.

preferences as

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t]$$

where $X_t \in \mathbb{R}$ is consumption in the *CM*, and $h_t \in \mathbb{R}_+$ is labor supply in the *DM*. Buyers can produce in the *CM*, but cannot produce in the *DM* while sellers can produce in the *DM*, but cannot produce in the *CM*. One unit of labor inputs can produce one unit of consumption goods in the *CM* and *DM*. It is assumed that the consumption goods are perfectly divisible and perishable.

There are two types of nominal government-issued assets. Fiat money trades at price ϕ_t in terms of goods in the competitive market of period t *CM*. One-period maturity government bonds is an obligation to pay one unit of fiat money in the period $t + 1$ *CM*. The price of government bonds is z_t in terms of goods in the period t *CM*. There are also two types of real private assets - two divisible Lucas trees. Each tree is endowed to buyers in the initial period *CM* with a fixed unit supply. One tree, named as liquid tree, pays dividend y^l and trades at the price ψ_t^l in terms of goods in period t *CM* whereas the other tree, called as illiquid tree pays dividend y^i and trades at the price ψ_t^i in terms of goods in period t *CM*.⁴

In the beginning of the *CM*, buyers and sellers meet together and debts are settled. Buyers receive lump-sum transfer or pay lump-sum tax and the holders of the Lucas trees receive the realized dividends. A Walrasian market opens, assets and goods are traded competitively. In the *DM* each buyer meets each seller bilaterally and the terms of trade are determined by bargaining. The buyer make a take-it-or-leave-it offer to the seller in the meetings. I assume that all agents are anonymous and there is no public record-keeping technology in the *CM* and *DM*. Thus recognizable assets are essential for trade and all trade must be *quid pro quo*. Similar to Sanches and Williamson (2010), there are two types of meetings by the types of buyers in the *DM*. In a fraction ρ of *DM* meetings with type 1

⁴This Lucas tree represents private investment as described in Lagos and Rocheteau (2008), but since the supply is fixed the inefficiency is reflected to its price instead of quantity as shown in Lagos and Rocheteau (2008).

buyers, fiat money and the liquid tree are the only assets recognized by sellers. In $1 - \rho$ fraction of *DM* meetings with type 2 buyers, the sellers can verify the entire portfolio held by buyers.⁵ Limited commitment is assumed so that agents can run away in the next *CM*, but the assets are seized and settled as a collateral. In sum type 1 buyers can trade only with fiat money or liquid tree and have a utility with $\theta_t^1 = 1$ in the *DM* whereas type 2 buyers can trade through credit arrangement with all their asset portfolio and have a utility with $\theta_t^2 = \theta$.

Given idiosyncratic liquidity shock a banking arrangement arises endogenously to allocate liquid and illiquid assets by the type of buyers efficiently. Without a banking contract type 2 buyers could hold idle liquid assets while type 1 buyers could hold idle illiquid assets. All agents can propose a banking contract which provides liquidity insurance in a way of Diamond and Dybvig (1983). It is socially optimal that banks provide liquid assets including fiat money to type 1 buyers who move to ρ proportion of meetings and provide illiquid assets including government bonds to type 2 buyers who move to $1 - \rho$ proportion of meetings. Banks observe the size of shock ρ exactly, but banks cannot verify the type of buyers after realization. Thus one type of buyers can mimic the other type of buyers under private information.⁶ In this environment if the real value of liquid assets to type 1 buyers is greater than the value of illiquid assets to type 2 buyers then type 2 buyers will mimic type 1 buyers since liquid assets are also useful for trade in $1 - \rho$ proportion of meetings. To support banking arrangement I assume that buyers can meet only a bank in the *CM* after their liquidity shock is realized. If *ex post* asset-trading among buyers is allowed then banking contract is unraveled and collapsed as shown in Jacklin (1987).⁷

⁵It is not contrary to no record-keeping technology assumption. Suppose sellers accept a deposit claim issued by banks and backed by asset portfolio of buyers, and redeem it in the next *CM*.

⁶In order to avoid bank runs created by sequential service I assume that buyers send a notification about their types to the bank simultaneously.

⁷Note that a banking contract equilibrium provides higher welfare than assets-trading market equilibrium. It is because a bank contract can provide resources to high marginal utility agents as much as it can regardless of prices.

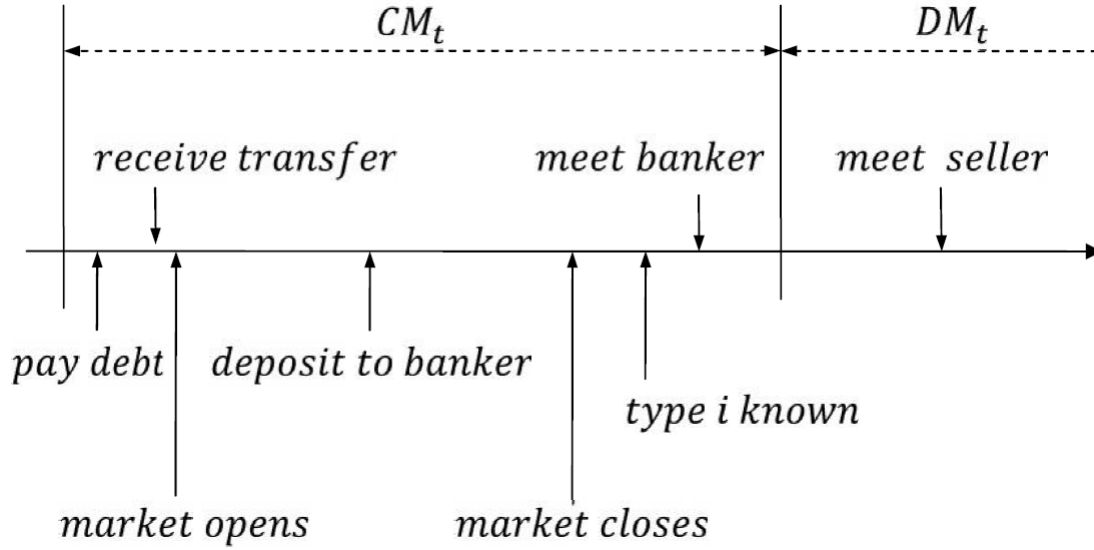


Figure 2.1: Time line

Timing is as follows as shown in Figure 2.1. In the beginning of CM deposit claims are paid and government-bonds holders can receive a unit of fiat money by redeeming a unit of government bonds. Then buyers receive lump-sum transfer(or pay lump-sum tax). All buyers provide labor and trade assets with sellers in a Walrasian market. Buyers deposit goods or assets into a banker with a banking contract. After liquidity shock is realized, buyers learn their types and ρ buyers meet the banker to withdraw currency and liquid assets. In the DM buyers meet sellers randomly in the bilateral meeting and make take-it-or-leave-it offers. In the next CM $1 - \rho$ sellers can redeem deposit claims to the banker or sell them to buyers.

2.2.1 Government

In the model government consists of fiscal authority and monetary authority. Fiscal authority can enforce lump-sum tax or provide transfer to buyers in the CM and issue government bonds and pay interests in the next CM . Monetary authority can issue fiat money and inject

or absorb fiat money by exchanging money with government bonds in the asset market, i.e. open-market-operations. I assume that private assets are not eligible to be an object for OMOs. Thus after government bonds are issued and fiat money is injected by open-market-operations at $t = 0$ and the revenue of issueing government bonds and fiat money is transferred to buyers. Then outstanding fiat money and government bonds can be supported by tax or transfer over time. So the consolidated government budget constraints are described as

$$\phi_0(M_0 + z_0 B_0) = \tau_0 = V$$

and

$$\phi_t\{M_t - M_{t-1} + z_t B_t - B_{t-1}\} = \tau_t, \quad t = 1, 2, 3, \dots$$

where M_t and B_t denote the nominal quantities of fiat money and government bonds held by private sector in the *CM* at time t , respectively. τ_t denote the real value of the lump-sum transfer from each buyer to the fiscal authority in the *CM* at period t . I assume that the fiscal authority keeps the total value of the outstanding consolidated government debt as a constant V after it is transferred with an exogenously fixed amount at $t = 0$. Thus in every period to maintain the real value of outstanding consolidated government debt, the real term of lump-sum transfer τ_t is derived passively from

$$\tau_t = \underbrace{\left(V_t - \frac{\phi_t}{\phi_{t-1}} V_{t-1}\right)}_{\text{seigniorage}} + \underbrace{\frac{\phi_t}{\phi_{t-1}} (1 - z_{t-1}) \phi_{t-1} B_{t-1}}_{\text{real interest payment}}, \quad t = 1, 2, 3, \dots,$$

Note that the lump-sum transfer consists of seigniorage from inflation and real interest payment for government bonds. The fixed real value of consolidated government debt assumption allows us to separate monetary policy, specifically OMOs, from fiscal policy. Moreover, when V is assumed as small enough, assets can be insufficient to support the optimal level

of consumption.

2.3 Competitive Equilibrium with Liquid and Illiquid Assets

In this section to emphasize private information friction I assume that illiquid assets are not useful for trade at all with $\theta = 0$. Also, in order to make a clear point I exclude the government liabilities by assuming $V = 0$. So we have only two different Lucas trees in the model. One is useful for trade with price ψ_t^l and dividend y^l while the other cannot be used in transactions at all with price ψ_t^i and dividend y^i . In the following subsections I consider each perfect information and private information case and compare the equilibrium allocations to understand the reason why the liquidity insurance can be restricted by the private information. Under private information banks suggest two type-dependent consumption offers $\{(x_1^l, x_1^i), (x_2^l, x_2^i)\}$ for type 1 and type 2 buyers to reveal their types. Note that superscripts denote type of assets between *liquid* and *illiquid* while subscripts denote type j of buyers: x_{jt}^l, x_{jt}^i represents the consumption level of type j buyers with liquid and illiquid assets. Under perfect competition a representative bank suggests a banking contract to maximize buyers' ex ante expected value. In equilibrium the bank solves the following generalized problem in the CM of period t :

$$\underset{d_t, a_t^l, a_t^i, x_{1t}^l, x_{1t}^i, x_{2t}^l, x_{2t}^i}{Max} - d_t + \rho\{u(x_{1t}^l) + x_{1t}^i\} + (1 - \rho)\{x_{2t}^l + x_{2t}^i\} \quad (2.1)$$

subject to a participation constraint of the bank,

$$d_t - \psi_t^l a_t^l - \psi_t^i a_t^i + \{\beta(\psi_{t+1}^l + y^l)a_t^l - \rho x_{1t}^l - (1 - \rho)x_{2t}^l\} + \{\beta(\psi_{t+1}^i + y^i)a_t^i - \rho x_{1t}^i - (1 - \rho)x_{2t}^i\} \geq 0 \quad (2.2)$$

and incentive constraints of the bank,

$$\beta(\psi_{t+1}^l + y^l)a_t^l - \rho x_{1t}^l \geq 0 \quad (2.3)$$

$$\beta(\psi_{t+1}^l + y^l)a_t^l + \beta(\psi_{t+1}^i + y^i)a_t^i - \rho x_{1t}^i - (1 - \rho)\{x_{2t}^l + x_{2t}^i\} \geq 0 \quad (2.4)$$

and truth-telling constraints,

$$u(x_{1t}^l) + x_{1t}^i \geq u(x_{2t}^l) + x_{2t}^i \quad (2.5)$$

$$x_{2t}^l + x_{2t}^i \geq x_{1t}^l + x_{1t}^i \quad (2.6)$$

and non-negative constraints,

$$d_t, a_t^l, a_t^i, x_{1t}^l, x_{1t}^i, x_{2t}^l, x_{2t}^i \geq 0 \quad (2.7)$$

All quantities in equations (2.1)-(2.7) are expressed in units of the *CM* good in time t . The problem (2.1) subject to constraints (2.2)-(2.7) states that a banking contract is chosen in equilibrium to maximize the expected utility of the representative buyer subject to the participation constraint for the banks (2.2) and liquid asset constraint (2.3) and collateral constraint (2.4) and individual incentive constraints by types (2.5)-(2.6) and non-negativity constraints (2.7). In (2.1)-(2.7) d_t denotes deposit of buyers, a_t^l, a_t^i denote demand for liquid and illiquid asset holdings of banks, ψ_t^l, ψ_t^i denote the prices of liquid and illiquid assets in the *CM*, respectively. The quantity on the left side of (2.2) is the net payoff for banks. In the *CM* of time t the banks receive d_t deposits and invest in liquid and illiquid assets with market prices then the banks pay x_{jt}^l to each type buyer before the *DM* and pay x_{jt}^i to the holders of deposit claims in the following *CM*. The participation constraint (2.2) implies that when deposit claims are paid off, the net payoff for the banks must be greater than zero. The liquid asset constraint (2.3) implies that type 1 buyers can withdraw liquid assets

by the limit of liquid asset holdings. The collateral constraint (2.4) implies that the liquid and illiquid assets can be seized when the bank decides to abscond. Individual incentive constraint (2.5)-(2.6) represent that each type of buyer weakly prefer an offer for own type to the offer for other type after type shock is realized. Note that type 1 buyers can also consume with illiquid assets in the next *CM*.

2.3.1 Perfect Information

For a benchmark I consider competitive equilibrium with perfect information. In case of perfect information banks know the buyer's type exactly after the liquidity shock is realized. In equilibrium *ex post* banks will provide all of liquid assets to type 1 buyers who only can trade in the *DM*. Since illiquid assets are useless for trade banks will not hold these assets as long as the real return of the illiquid asset are less than time preference. If the real return of the illiquid asset is same as time preference then banks can hold these illiquid assets and provide them to type 1 or 2 buyers, but it is irrelevant since both type buyers have linear utility function for illiquid assets. Thus without loss of generality I assume that banks do not hold illiquid assets, i.e. $x_{1t}^i = x_{2t}^i = 0$, with perfect information. Moreover, truth-telling constraints are unnecessary. Thus given price ψ_t^l , a representative bank solves the reduced maximization problem in the *CM* of period t :

$$\underset{d_t, a_t^l, x_{1t}^l}{Max} -d_t + \rho u(x_{1t}^l) \quad (2.8)$$

subject to the participation constraint,

$$d_t - \psi_t^l a_t^l + \{\beta(\psi_{t+1}^l + y^l)a_t^l - \rho x_{1t}^l\} \geq 0 \quad (2.9)$$

and liquid asset constraint,

$$\beta(\psi_{t+1}^l + y^l)a_t^l - \rho x_{1t}^l \geq 0 \quad (2.10)$$

and non-negative constraints,

$$d_t, a_t^l, x_{1t}^l \geq 0 \quad (2.11)$$

By plugging (9) into (8) we have the first-order conditions by a_t^l, x_{1t}^l ,

$$\psi_t^l = \beta(\psi_{t+1}^l + y^l)(1 + \lambda_t) \quad (2.12)$$

$$u'(x_{1t}^l) - 1 = \lambda_t \quad (2.13)$$

where λ_t is a multiplier associated with liquid asset constraint (10). The first-order conditions (12) and (13) can be reduced to

$$\psi_t^l = \beta(\psi_{t+1}^l + y^l)u'(x_{1t}^l) \quad (2.14)$$

In equilibrium asset market clear in the *CM* and a representative bank holds all the liquid asset in its portfolio for $t = 0, 1, 2, \dots$. The supply of liquid asset is equal to the demand of banks as

$$a_t^l = 1. \quad (2.15)$$

Definition 2.1 : *Given (ρ, y^l, y^i) a stationary competitive equilibrium under perfect information consists of quantity x_{1t}^l and price ψ_t^l and multiplier λ which solves equations (2.10), (2.14), (2.15).*

In what follows I focus on stationary equilibrium allocations without time scripts on variables. There are two equilibrium cases following by the value of y .

Case (i) Suppose that the liquid asset constraint (2.10) does not bind. That means, in equilibrium the first-best consumption level, x^* where $u'(x^*) = 1$, is achieved for type 1

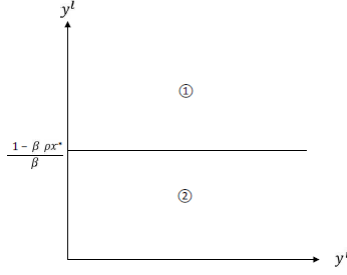


Figure 2.2: Perfect Information

buyers. From the first-order condition (2.14), the asset price is the same as its fundamental value: $\psi^l = \psi_f^l$ holds where $\psi_f^l \equiv \frac{\beta y^l}{1-\beta}$. Note that this case of equilibrium is supported by $y^l \geq \frac{(1-\beta)}{\beta} \rho x^*$ from (2.10).

Case (ii) Suppose that the liquid asset constraint (2.10) binds with $y^l < \frac{(1-\beta)}{\beta} \rho x^*$. Then the equilibrium allocation (x_1^l, ψ^l) is uniquely determined from (2.10) and (2.14) since ψ^l is strictly increasing in x_1^l from (2.10) while ψ^l is strictly decreasing in x_1^l from (2.14). Note that the consumption level is less than its optimal level, $x_1^l < x^*$ and the asset price is greater than its fundamental value, $\psi^l > \psi_f^l$ in equilibrium. Liquidity premium, the difference between the asset price and its fundamental value is strictly positive because of liquid asset shortage. The price of illiquid asset is same as its fundamental value as $\psi^i = \psi_f^i$ where $\psi_f^i \equiv \frac{\beta y^i}{1-\beta}$, if it is traded in the market.

These two cases can be described in Figure 2.2. If the supply of liquid asset is large enough with $y^l \geq \frac{(1-\beta)}{\beta} \rho x^*$ then we have the case 1 equilibrium with the first-best allocation. If the supply of liquid asset is low with $y^l < \frac{(1-\beta)}{\beta} \rho x^*$ then we would have this case 2 equilibrium.

2.3.2 Private Information

In case of private information as described above, banks solve the original maximization problem (2.1)-(2.7). To simplify the problem I use some lemmas here.

Lemma 2.2. (*Single Crossing Property*) *In equilibrium with $x_{1t}^l \in [0, x^*)$, both truth-telling*

constraints do not bind simultaneously.

proof If both truth-telling constraints (2.5) and (2.6) bind then $u(x_{1t}^l) - x_{1t}^l = u(x_{2t}^l) - x_{2t}^l$.

Since $u(x) - x$ is strictly increasing in $x \in [0, x^*]$, $x_{1t}^l = x_{2t}^l$ and $x_{1t}^i = x_{2t}^i$. Note that $x_{1t}^i = x_{2t}^i > 0$ for $y > 0$ in equilibrium. For $x_{1t}^l = x_{2t}^l < x^*$ the expected value of buyers can increase by transferring liquid assets from type 2 buyers to type 1 buyers and transferring the same amount of illiquid assets from type 1 buyers to type 2 buyers.

Contradiction. ■

Lemma 2.3. *In equilibrium with $x_{1t}^l \in [0, x^*)$, the truth-telling constraint for type 1 buyers does not bind.*

proof Suppose that the constraint (2.5) binds while the constraint (2.6) does not bind. If

$x_{2t}^l > 0$ then the expected value of buyers can increase by transferring liquid assets from type 2 buyers to type 1 buyers. If $x_{2t}^l = 0$ then the expected value of buyers are indifferent when illiquid assets are transferred from type 2 buyers to type 1 buyers that means (2.6) does not bind. Contradiction ■

In equilibrium the truth-telling constraint for type 1 buyers (2.5) does not bind. It is because banks allocate resources to type 1 buyers who have higher marginal utility as much as possible to maximize the expected utility for agents. Thus the incentive constraint for type 2 buyers always binds. In Figure 2.3, indifference curve of type 1 buyers intersects the indifference curve of type 2 buyers at x_{1t}^l . To keep utility for type 2 buyers x_{2t}^i is required. Since (2.5) does not bind, there is no difference between x_{2t}^i and x_{2t}^l in the problem so that they can be merged as x_{2t} . Note that type-dependent contract is non-linear in general, but in this case deposit contract is linear as standard deposit contract because quasi-linear utility is adopted.

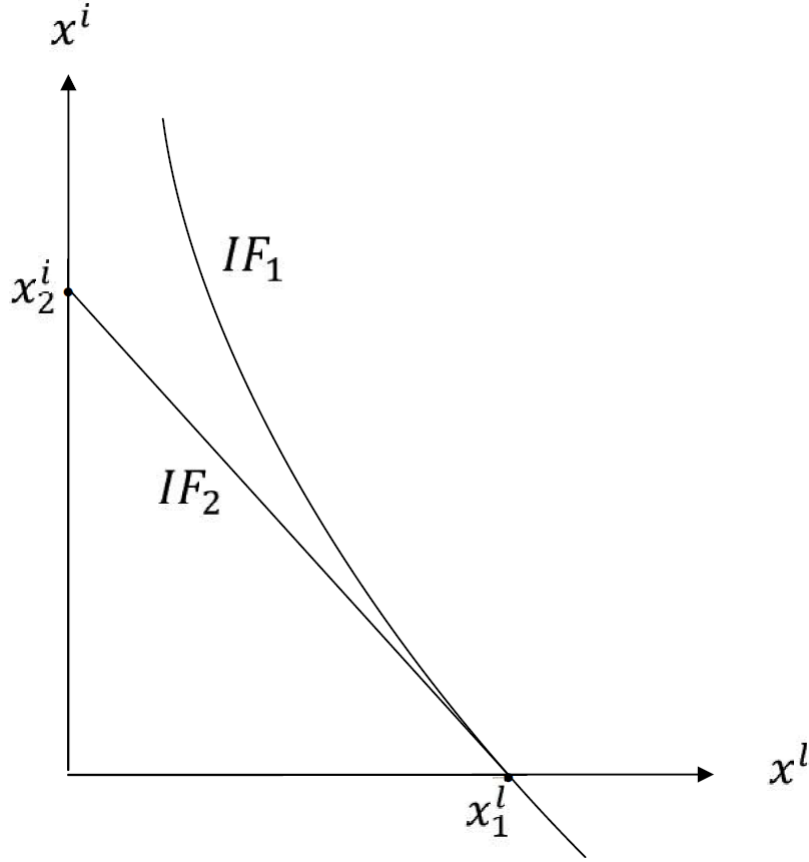


Figure 2.3: Single Crossing Property

Lemma 2.3. *In equilibrium with $x_{1t}^l \in [0, x^*)$, $x_{1t}^i = 0$.*

proof Suppose that $x_{1t}^i > 0$ in equilibrium. If the truth-telling constraint for type 2 buyers (2.6) binds then transferring illiquid assets to type 2 buyers can relax (2.6). If (2.6) does not bind then the expected value of buyers is indifferent. Thus there is no reason to have $x_{1t}^i > 0$ in equilibrium ■

Illiquid assets for type 1 buyers are unnecessary because neither it is used for trade nor it overcomes private information friction in (2.12). Since $x_{1t}^i = 0$ in equilibrium we can rename x_{1t}^l as x_{1t} . Without the truth-telling constraint for type 1 buyers (2.5) and with choice variable x_{1t} and x_{2t} , we have the first-order conditions by $a_t^l, a_t^i, x_{1t}, x_{2t}$,

$$\psi_t^l = \beta(\psi_{t+1}^l + y^l)(1 + \lambda_{1t} + \lambda_{2t}) \quad (2.16)$$

$$\psi_t^i = \beta(\psi_{t+1}^i + y^i)(1 + \lambda_{2t}) \quad (2.17)$$

$$\rho u'(x_{1t}) - \rho - \rho\lambda_{1t} - \lambda_{3t} = 0 \quad (2.18)$$

$$(1 - \rho) - (1 - \rho) - (1 - \rho)\lambda_{2t} + \lambda_{3t} = 0 \quad (2.19)$$

where $\lambda_{1t}, \lambda_{2t}$ and λ_{3t} denote each multiplier associated with the constraints (2.3), (2.4) and (2.6). In equilibrium asset markets clear in the *CM* and a representative bank holds all the liquid and illiquid assets in its portfolio for $t = 0, 1, 2, \dots$. The supply of liquid and illiquid asset is equal to its demand from banks, respectively, as shown in (2.20).

$$a_t^l = a_t^i = 1 \quad (2.20)$$

Definition 2.4. *Given (ρ, y^l, y^i) a stationary competitive equilibrium under private information consists of quantity x_1, x_2 and price ψ_t^l, ψ_t^i and multiplier $\lambda_{1t}, \lambda_{2t}, \lambda_{3t}$ which solves equations (2.3)-(2.4), (2.6), (2.16)-(2.20).*

I focus on stationary equilibrium allocations without time scripts on variables. Note that the collateral constraint (2.4) and the truth-telling constraint (2.6) either binds or does not bind together from (2.19) because $x_2 > 0$ in equilibrium with $y^i > 0$. If the truth-telling constraint (2.6) binds with $\lambda_3 > 0$ then the liquid asset constraint (2.3) is relaxed with $\lambda_1 = 0$ because only one of them can restrict $x_1 \in [0, x^*)$ in equilibrium. In sum under private information there are three equilibrium cases. As discussed in perfect information subsection we have case (i) and (ii) equilibrium and additionally a new equilibrium, case (iii)

equilibrium, in which the truth-telling constraint (2.6) binds and the liquid asset constraint (2.3) is relaxed.

Case (i) When the constraints (2.3), (2.4), (2.6) do not bind we have

$$\frac{\psi^l}{\beta(\psi^l + y^l)} = \frac{\psi^i}{\beta(\psi^i + y^i)} = 1 \quad (2.21)$$

and

$$1 = u'(x_1) \quad (2.22)$$

from the first-order conditions (2.16)-(2.18). Since the constraints do not bind, the optimal level of consumption is achieved, $x_1 = x^*$, for type 1 buyers and the asset prices are the same as their fundamental values, $\psi^l = \psi_f^l$, $\psi^i = \psi_f^i$ from (2.21)-(2.22). The equilibrium is supported by a region with $\psi_f^l \geq \rho x^*$ and $\psi_f^l + \psi_f^i \geq x^*$ from (2.3)-(2.4), (2.20). Note that this first-best equilibrium allocation is the same as one in perfect information case (i). One interesting point is that this first-best equilibrium allocation is feasible even when there exists a degree of private information as long as liquid and illiquid assets are plentiful in the economy.

Case (ii) When the liquid asset constraint (2.3) binds only, we have

$$\frac{\psi^i}{\beta(\psi^i + y^i)} = 1 \quad (2.23)$$

and

$$\frac{\psi^l}{\beta(\psi^l + y^l)} = u'(x_1) \quad (2.24)$$

from (2.16)-(2.18). The binding constraint (2.3) with asset market clear condition (2.20) can be reduced into

$$\beta(\psi^l + y^l) = \rho x_1 \quad (2.25)$$

Then (x_1, ψ^l) are uniquely determined from (2.24) and (2.25). In equilibrium we have $x_1 < x^*, \psi^l > \psi_f^l$, $\psi^i = \psi_f^i$ and $x_1 \leq x_2$. Note that a liquidity premium arises in the price of the liquid asset since the inefficiency is caused by the scarcity of liquid asset as described in Champ, Smith and Williamson (1996). On the other hand, the price of illiquid asset keeps at its fundamental value because those illiquid assets are already plentiful. The equilibrium is supported by a region which satisfies with $\psi_f^l < \rho x^*$ and $y^l \geq \frac{\rho}{1-\rho} y^i$. If y^i is too low then there exists a threshold point in which the truth-telling constraint (2.6) starts to bind while the liquid asset constraint (2.3) is just relaxed. In this threshold point we have $\beta(\psi^l + y^l) = \rho x_1$, $\beta(\psi^i + y^i) = (1-\rho)x_2$, $x_2 = x_1$ from (2.3), (2.4), (2.6). We also have $\frac{\psi^i}{\beta(\psi^i + y^i)} = \frac{\psi^l}{\beta(\psi^l + y^l)}$ from (2.16)-(2.17) since the liquid asset constraint (2.3) is just slack at the point. Note that those conditions can be reduced into $y^l = \frac{\rho}{1-\rho} y^i$ which is a threshold borderline between case (ii) and case (iii) equilibrium. Note that this case (ii) equilibrium is also the same as the case 2 equilibrium under perfect information.

Case (iii) When the collateral constraint (2.4) and the truth-telling constraint (2.6) bind, we have

$$\frac{\psi^l}{\beta(\psi^l + y^l)} = \frac{\psi^i}{\beta(\psi^i + y^i)} = \frac{\{\rho u'(x) + 1 - \rho\} - \rho}{1 - \rho} \quad (2.26)$$

and

$$\beta(\psi^l + y^l) + \beta(\psi^i + y^i) = x \quad (2.27)$$

from (2.16)-(2.20) where $x \equiv x_1 = x_2$ is denoted. Then (x, ψ^l, ψ^i) are uniquely determined from the equilibrium condition (2.26)-(2.27). In equilibrium we have $x < x^*, \psi^l > \psi_f^l$, $\psi^i > \psi_f^i$. Note that a liquidity premium arises in the both prices of liquid and illiquid assets

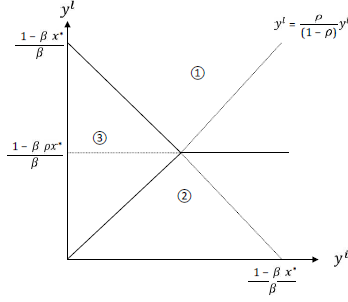


Figure 2.4: Private Information

although the liquid asset constraint does not bind in equilibrium. It is because both liquid and illiquid assets are scarce to reveal the private information in equilibrium. In equilibrium a proportion of liquid assets must be provided to type 2 buyers so that rates of return in liquid asset and illiquid asset are same in equilibrium. Note that a liquidity premium arises although illiquid assets are not useful for trade at all.

Three regions of equilibrium under private information are described in Figure 2.4. Notice that the first-best equilibrium allocation appears when liquid assets are plentiful and are supported by sufficient illiquid assets. The case (ii) equilibrium allocation is shown when liquid assets are scarce although those liquid assets are supported by illiquid assets enough. The case (iii) equilibrium allocation arises when the types are hardly revealed because of the scarcity of illiquid assets.

2.4 Monetary Equilibrium

In this section I verify how this private information restricts the implementation of monetary policy, specifically open-market-operations. We have money and government bonds in the model with $V > 0$, but do not have private liquid assets with $y^l = 0$. Note that private liquid assets are irrelevant because we focus on the case in which illiquid assets are scarce. Also type 2 buyers can consume through credit arrangement. However, the marginal utility in credit arrangement is less than the marginal utility in currency trade with $\theta \in (0, 1)$.

This assumption represents that credit arrangement trade is less preferable than currency trade in a view of social welfare. One rationalization of this assumption is a social cost of operating payment and settlement for credit arrangement. Note that this assumption will provide an incentive for banks to increase currency exchange more than credit arrangement for social optimality. Thus this assumption lets the truth-telling constraint bind when the bank maximizes the ex ante expected value of buyers. Note that the model can be reduced to the baseline model of Williamson (2014) if type information is perfect and $\theta = 1$. A representative bank solves the following modified problem in the *CM* of period t :

$$\underset{d_t, m_t, b_t, a_t, x_{1t}, x_{2t}}{Max} -d_t + \rho u(x_{1t}) + (1 - \rho)\theta u(x_{2t}) \quad (2.28)$$

subject to the participation constraint,

$$d_t - m_t - z_t b_t - \psi_t^i a_t^i + \left\{ \frac{\beta \phi_{t+1}}{\phi_t} m_t - \rho x_{1t} \right\} + \left\{ \frac{\beta \phi_{t+1}}{\phi_t} b_t + \beta(\psi_{t+1}^i + y^i) a_t^i - (1 - \rho)x_{2t} \right\} \geq 0 \quad (2.29)$$

and the cash constraint,

$$\frac{\beta \phi_{t+1}}{\phi_t} m_t - \rho x_{1t} \geq 0 \quad (2.30)$$

and the collateral constraint,

$$\frac{\beta \phi_{t+1}}{\phi_t} (m_t + b_t) + \beta(\psi_{t+1}^i + y^i) a_t^i - \rho x_{1t} - (1 - \rho)x_{2t} \geq 0 \quad (2.31)$$

and the truth-telling constraint,

$$x_{2t} \geq x_{1t} \quad (2.32)$$

and non-negative constraints,

$$d_t, m_t, b_t, a_t^i, x_{1t}, x_{2t} \geq 0 \quad (2.33)$$

The problem (2.28) subject to the constraints (2.29)-(2.33) states that a banking contract is chosen in equilibrium to maximize expected utility of the buyers subject to participation constraint (2.29) in which banks receive a non-negative profit by providing the contract and cash constraint for type 1 buyers (2.30) and collateral constraint for type 2 buyers (2.31) and the incentive constraint for type 2 buyers (2.32) and non-negative constraints (2.33). I omitted the incentive constraint for type 1 buyers because it does not bind in equilibrium as shown in Lemma 2.2 and 2.3.⁸ In (2.28)-(2.33), d_t denotes deposit of buyers, a_t denote demand for illiquid asset holdings of banks, ψ_t denote the prices of illiquid assets in the CM, m_t and b_t denote the quantities of money and government bonds in terms of the CM good in period t held by banks and x_{jt} denote the consumption of type j agents at time t CM for $j \in \{1, 2\}$. From now on I focus on stationary equilibrium where $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ holds for all t , and μ is the gross inflation rate. Note that nominal interest rate of government bonds cannot be negative, i.e. $z_t \leq 1$, in equilibrium by its feasibility assumption. I assume that monetary authority sets the inflation rate target and implement OMOs to achieve its goal.

From the maximization problem the first-order conditions by $m_t, b_t, a_t, x_{1t}, x_{2t}$ can be described as follows.

$$\frac{\mu}{\beta} = 1 + \lambda_1 + \lambda_2 \quad (2.34)$$

$$z \frac{\mu}{\beta} = \frac{\psi}{\beta(\psi + y)} = 1 + \lambda_2 \quad (2.35)$$

$$u'(x_1) - \frac{\lambda_3}{\rho} = 1 + \lambda_1 + \lambda_2 \quad (2.36)$$

⁸Both individual incentive constraints do not change although type 2 buyers can trade since liquid and illiquid assets are used as same for collateral.

$$\theta u'(x_2) + \frac{\lambda_3}{1-\rho} = 1 + \lambda_2 \quad (2.37)$$

where λ_1 to λ_3 denote each multiplier associated with the constraints (2.30)-(2.32). In equilibrium asset markets clear in the CM with

$$a^i = 1 \quad (2.38)$$

$$m = \phi M \quad (2.39)$$

$$b = \phi B \quad (2.40)$$

Since the supply of government assets are restricted by the consolidated government debt limit V we have

$$m + zb \leq V \quad (2.41)$$

Definition 2.5. *Given (ρ, V, y) and the inflation rate target μ , a stationary monetary equilibrium consists of quantities (x_1, x_2) and prices (z, ψ) and multipliers $(\lambda_1, \lambda_2, \lambda_3)$ which solve equations (2.30)-(2.32), (2.34)-(2.37), (2.41).*

Since quasi-linear utility is adopted the real return of assets such as fiat money, $\mu = \frac{\phi_{t-1}}{\phi_t}$, and government bonds, $\frac{1}{z\mu}$, illiquid Lucas tree, $\frac{\psi^i + y^i}{\psi^i}$, cannot exceed the rate of time preference, $\frac{1}{\beta}$. The rate of returns in government bonds and Lucas tree are same in equilibrium because there is no credit risk in Lucas tree. Nominal interest rate of government bonds cannot be negative, i.e. $z \leq 1$, by its feasibility assumption. Then we have no arbitrage condition in equilibrium,

$$\frac{1}{\mu} \leq r \equiv \frac{1}{z\mu} = \frac{\psi^i + y^i}{\psi^i} \leq \frac{1}{\beta}. \quad (2.42)$$

Note that if the truth-telling constraint (2.32) binds then cash constraint (2.30) is relaxed because only one of them can restrict $x_1 \in [0, x^*)$ in equilibrium. Moreover, when the truth-telling constraint (2.32) binds collateral constraint (2.31) is required to bind, otherwise the truth-telling constraint (2.32) does not bind with $x_2 = x_2^*$. Hence when the truth-telling constraint (2.32) binds we have only one case of equilibrium with $\lambda_1 = 0, \lambda_2 > 0, \lambda_3 > 0$. On the other hand, if the truth-telling constraint (2.32) does not bind then we have four cases of equilibrium with combination of λ_1 and λ_2 . However, the case constraint (2.30) cannot be relaxed alone because the liquid assets can be also useful for credit arrangement. Thus we have totally four equilibrium cases.

(1) Friedman rule Equilibrium When all of the constraints (2.30)-(2.32) do not bind with $\lambda_1 = \lambda_2 = \lambda_3 = 0$, we have

$$\frac{\mu}{\beta} = z \frac{\mu}{\beta} = \frac{\psi^i}{\beta(\psi^i + y^i)} = 1 \quad (2.43)$$

$$1 = u'(x_1) = \theta u'(x_2) \quad (2.44)$$

from the first-order conditions (2.34)-(2.37). Note that $x_1 = x_1^*, x_2 = x_2^*$ and $\psi^i = \psi_f^i$ where $u'(x_1^*) = 1, \theta u'(x_2^*) = 1, \psi_f^i = \frac{\beta y^i}{1-\beta}$ in equilibrium. Moreover, the rates of return in both liquid and illiquid assets are the same as the inverse of time preference, $\frac{1}{\mu} = r = \frac{1}{\beta}$ where $r \equiv \frac{1}{z\mu} = \frac{\psi^i + y^i}{\psi^i}$ from (2.43)-(2.44) in equilibrium. Thus in this case the Friedman rule, $\mu = \beta$, is feasible so that the first-best allocation is achieved. The equilibrium is supported by a region with $V \geq \rho x_1^*$ and $V + \psi_f^i \geq \rho x_1^* + (1 - \rho)x_2^*$ from the equations (2.30)-(2.31), (2.38), (2.41). Note that this case is the same as the case (i) in competitive equilibrium in the previous section.

(2) Currency-shortage Equilibrium Suppose that the Friedman rule equilibrium is infeasible with $V < \rho x_1^*$, while $\psi_f^i \geq (1 - \rho)x_2^*$ is valid. With $\lambda_1 > 0, \lambda_2 = \lambda_3 = 0$ we have

the first-order conditions,

$$\frac{\mu}{\beta} = u'(x_1) \quad (2.45)$$

$$z \frac{\mu}{\beta} = \frac{\psi^i}{\beta(\psi^i + y^i)} = 1 = \theta u'(x_2) \quad (2.46)$$

in which $x_2 = x_2^*$ and $\psi^i = \psi_f^i$. Binding cash constraint (2.30) is transformed into $\rho x_1 u'(x_1) = V$. Thus $x_1 < x_1^*$ is fixed in equilibrium. Then in a region of $x_1 \in (0, x_1^*)$ we have $x_1 < x_2^*$ and $\frac{1}{\mu} < r = \frac{1}{\beta}$ in equilibrium. Define this case as *Currency-shortage Equilibrium*. In this case real interest rate of illiquid asset is fixed and a liquidity premium arises in the price of money since only money is scarce. Open market operations are ineffective in real allocations since real interest rate of illiquid assets and x_1 are fixed in equilibrium. Note that this case is the same as the case (ii) in competitive equilibrium in the previous section.

(3) Asset-shortage Equilibrium Suppose that the Friedman rule equilibrium is infeasible with $V + \psi_f^i < \rho x_1^* + (1 - \rho)x_2^*$, but $V \geq \rho x_1^*$ is still valid. Moreover, assume that $\lambda_3 = 0$. Then we have the first-order conditions,

$$\frac{\mu}{\beta} = u'(x_1) \quad (2.47)$$

$$z \frac{\mu}{\beta} = \frac{\psi^i}{\beta(\psi^i + y^i)} = \theta u'(x_2) \quad (2.48)$$

from (2.34)-(2.37). Binding constraints (2.30) and (2.31) with asset market clearing condition (2.38) and government budget constraint (2.41) can be reduced into

$$\rho x_1 u'(x_1) + (1 - \rho) \theta x_2 u'(x_2) = V + \frac{\beta y^i \theta u'(x_2)}{1 - \beta \theta u'(x_2)} \quad (2.49)$$

Given the price of government bonds z , x_1 and x_2 are positively related in (2.47) and (2.48) while x_1 and x_2 is negatively related in (2.49). Thus there exists a unique equilibrium allocation (x_1, x_2) as shown in Figure 2.5. Let me briefly describe threshold points as shown in Figure 2.5. There is a threshold point \hat{x}_1 at $x_2 = x_2^*$ in which the collateral constraint starts to bind. There is another point \tilde{x}_1 in which $x_1 = x_2$ holds with the nominal interest rate $z = \theta$. Finally, there is a point \bar{x}_1 where the equilibrium allocation is determined with the zero nominal interest rate, $z = 1$. Through open market operations, the monetary authority can inject money and absorb government bonds in the market. By conducting this procedure the currency trade x_1 increases whereas the credit arrangement x_2 decreases. Thus the monetary authority can choose an equilibrium allocation in $x_1 \in (\hat{x}_1, \bar{x}_1]$ by choosing the price of government bonds z in equilibrium. However, in this private information case the truth-telling constraint matters when x_1 becomes greater than x_2 . Thus this asset-shortage equilibrium can exist only in $x_1 \in (\hat{x}_1, \tilde{x}_1]$ with $z \leq \theta$.

(4) Liquidity-trap Equilibrium Given the Friedman rule equilibrium is infeasible with $V + \psi_f^i < \rho x_1^* + (1 - \rho)x_2^*$ and $V \geq \rho x_1^*$, suppose that truth-telling constraint (32) binds with $\lambda_3 > 0$. As discussed when (32) binds we have only one case of equilibrium with $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 > 0$. The first-order conditions are reduced into

$$\frac{\mu}{\beta} = z \frac{\mu}{\beta} = \frac{\psi^i}{\beta(\psi^i + y^i)} = u'(x_1) - \frac{\lambda_3}{\rho} = 1 + \lambda_2 \quad (2.50)$$

$$\theta u'(x_2) + \frac{\lambda_3}{1 - \rho} = 1 + \lambda_2 \quad (2.51)$$

from (2.34)-(2.37). Since the truth-telling constraint binds, we have $x \equiv x_1 = x_2$ in equilibrium. Thus (2.50) and (2.51) can be reduced into

$$u'(x) - \frac{\lambda_3}{\rho} = \theta u'(x) + \frac{\lambda_3}{1 - \rho} \quad (2.52)$$

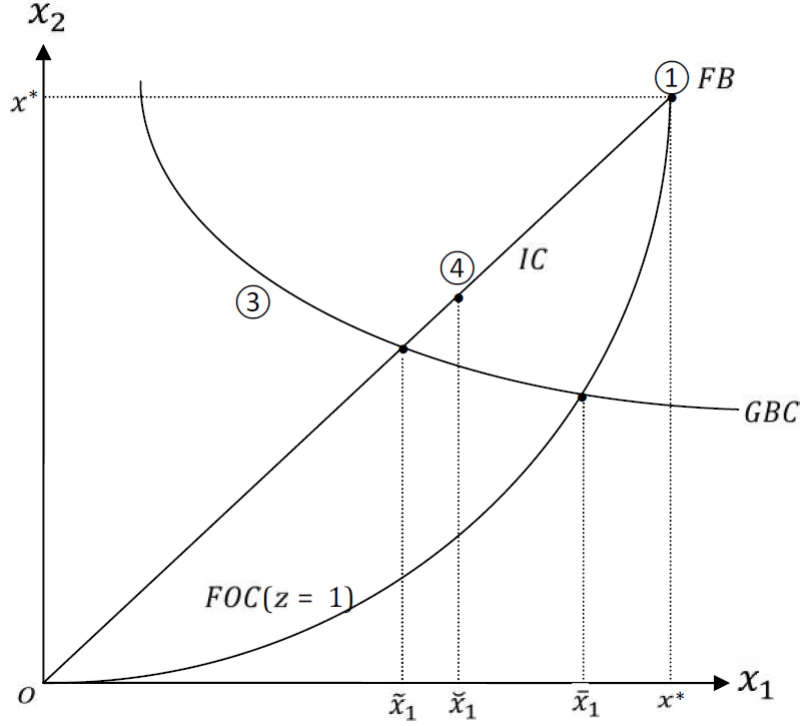


Figure 2.5: Liquidity Trap Equilibrium

Then binding constraints (2.31)-(2.32) with asset market clearing condition (2.38) and government budget constraint (2.41) can be reduced into

$$\rho x u'(x) + (1 - \rho) \theta x u'(x) = V + \frac{\beta y^i \{ \rho u'(x) + (1 - \rho) \theta u'(x) \}}{1 - \beta \{ \rho u'(x) + (1 - \rho) \theta u'(x) \}} \quad (2.53)$$

The allocation x is determined at $x = \check{x}_1$ in which government budget constraint intersects with 45 degree line as shown in Figure 2.5. Note that \check{x}_1 is fixed and $\frac{1}{\mu} = r < \frac{1}{\beta}$ holds with $z = 1$ in equilibrium. Thus in the *Liquidity-trap equilibrium* rates of return in money and illiquid assets are same and open market operations are no longer effective. Note that the feasibility condition (2.53) is similar to (2.49) with $x_1 = x_2$. However, the consumption level x in (2.53) is greater than the consumption level $x_1 = x_2$ in (2.49) with $z = \theta$ because liquid assets also have a liquidity premium in the liquidity trap equilibrium.

2.4.1 Liquidity Trap and Excess Reserves

In this subsection let me elaborate the implementation of monetary policy in the *Asset-shortage Equilibrium* and *Liquidity-trap equilibrium*. Given the scarcity of illiquid assets with $V + \psi_f^i < \rho x_1^* + (1 - \rho)x_2^*$ and $V \geq \rho x_1^*$, there exist two different regions in the equilibrium under private information. In a region of $x_1 \in (\hat{x}_1, \tilde{x}_1]$ we have $x_1 < x_1^*$, $x_2 < x_2^*$ and $\frac{1}{\mu} < r < \frac{1}{\beta}$ in the *Asset-shortage Equilibrium*. In equilibrium the monetary authority can choose the equilibrium allocation along with the feasibility condition (2.49) by exchanging outside currency and government bonds in the market. For example, injecting money and absorbing government bonds decreases the nominal interest rate, $\frac{1}{z} - 1$, and currency trade x_1 increases whereas credit arrangement x_2 decreases. On the other hand, in a point of $x_1 = \check{x}_1$ we have $x_1 < x_1^*$, $x_2 < x_2^*$ and $\frac{1}{\mu} = r < \frac{1}{\beta}$ in the *Liquidity-trap Equilibrium* as described.

Thus if the type information is public, then the equilibrium allocation $x_1 = \bar{x}_1$ with $z = 1$ is feasible. Note that in this equilibrium we have $x_2 < x_1$ and $\frac{1}{\mu} = r < \frac{1}{\beta}$. Let's define this equilibrium case as *Zero-lower-bound (ZLB) Equilibrium*. In the *Zero-lower-bound equilibrium* rates of return in money and illiquid assets are also same as shown in *Liquidity-trap equilibrium*. Thus open market operations are also ineffective in real allocations. Injecting money just increases the amount of excess reserves and the nominal interest rate is zero in equilibrium. However, the equilibrium allocations are different between *Zero-lower-bound equilibrium* and *Liquidity-trap equilibrium*. Moreover, the reasons for the ineffective monetary policy are different. In the *Zero-lower-bound equilibrium*, monetary policy is ineffective because the rates of return are set as same for both liquid and illiquid assets. But in the *Liquidity-trap equilibrium* monetary policy is ineffective because the excess reserves are required to separate the types under private information. Thus the same rates of return on liquid and illiquid assets with zero nominal interest rate is a consequence of equilibrium allocation instead of choice of monetary authority.

Proposition 2.6 *Given $\theta < 1$, there exists a unique Liquidity-trap equilibrium away from the Zero-lower-bound equilibrium under private information.*

Proof. In Liquidity-trap equilibrium the allocation $x = x_1 = x_2$ which satisfies (2.53). It is unique since the left side of (2.53) is strictly increasing in x while the right side of (2.5) is strictly decreasing in x . The allocation $x = x_1 = x_2$ which satisfies (2.53) is different from the allocation in the Zero-lower-bound equilibrium because $x_1 > x_2$ holds in the Zero-lower-bound equilibrium.

Note that if $\theta = 1$ is assumed, *Liquidity-trap equilibrium* overlaps with *Zero-lower-bound equilibrium* because the truth-telling constraint does not bind even at the zero nominal interest rate.

It is hard to differentiate the existence of *Liquidity-trap equilibrium* from *Zero-lower-bound equilibrium* in reality. However, if there exists a cost of operating credit arrangement or inefficiency in credit arrangement such as haircut then there could exist a jump from $z = \theta$ to $z = 1$ in equilibrium.⁹ Figure 6, which is replicated from Orphanides (2004), describes a movement of nominal interest rates along with excess reserves in the period of Great Depression. It is shown that there exists a volatile movement in nominal interest rates with excess reserves in a period of Great Depression. This implies at least that the monetary authority could lose its control on nominal interest rates in a neighborhood of zero lower bound.

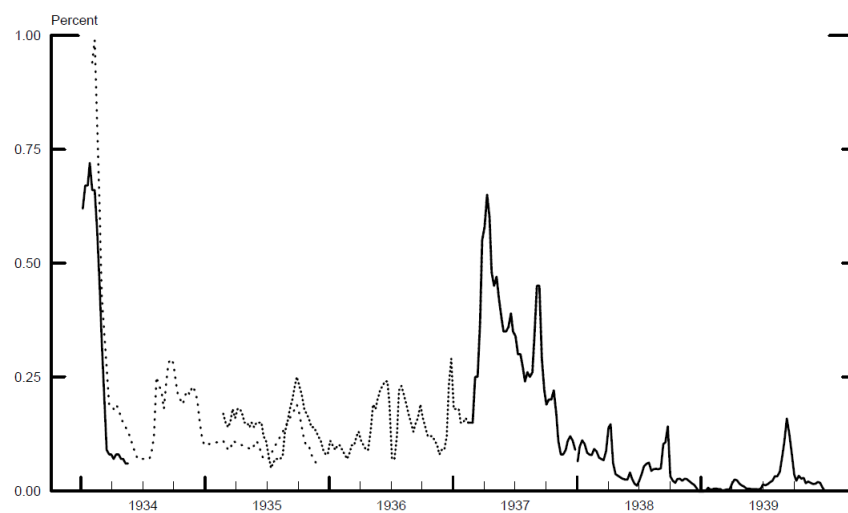
2.4.2 Private Assets

In this subsection I discuss whether the scarcity of private assets can influence in the equilibrium allocations when incentive constraint binds. If there is no private illiquid assets, $y^i = 0$, then the equilibrium conditions (2.49) and (2.53) are transformed into

$$\rho x_1 u'(x_1) + (1 - \rho)\theta x_2 u'(x_2) = V \quad (2.54)$$

⁹However, if a floor system which directly sets interest rates for reserves is available, then nominal interest rates are achieved exactly by setting the interest on reserves as same as the nominal interest rate target. Thus there would exist no jump in nominal interest rates.

Treasury Bill Rates



Excess Reserves

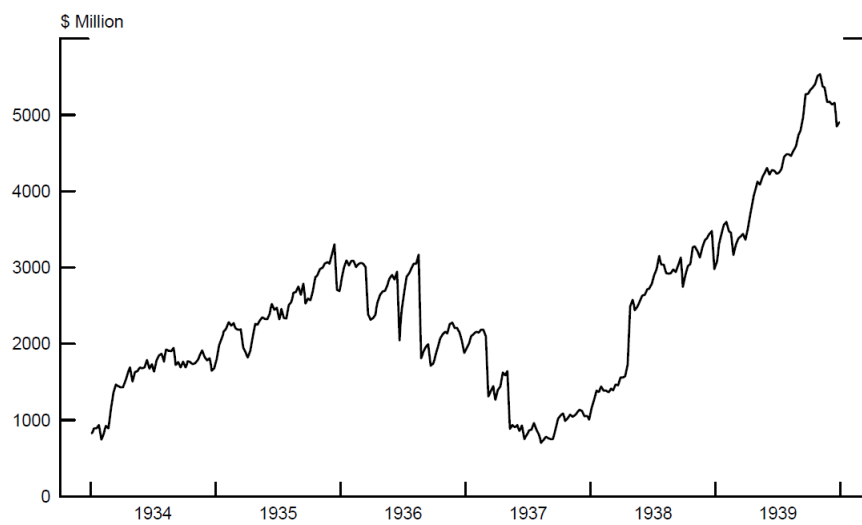


Figure 2.6: Treasury Bill Rates and Excess Reserves in Great Depression

and

$$\rho xu'(x) + (1 - \rho)\theta xu'(x) = V, \quad (2.55)$$

respectively. Then since (2.54) and (2.55) are same when the truth-telling constraint binds with $x_1 = x_2$, the equilibrium allocation does not change as excess reserves increase. However, if there is private illiquid assets, $y^i > 0$, then $x = \check{x}_1$ in (2.53) is greater than $x_1 = x_2 = \tilde{x}_1$ in (2.49) as long as $\theta < 1$. Thus there is a benefit of holding excess reserves. In the *Liquidity-trap equilibrium* the rate of return in liquid assets must be the same as the rate of return in illiquid assets because excess reserves are required in equilibrium. Thus rates of return in liquid assets would increase while rates of return in illiquid assets would decrease as equilibrium allocation moves from *Asset-shortage Equilibrium* to *Liquidity-trap equilibrium*. Thus the price of private assets would also increase and it would relax equilibrium condition (2.53).

2.4.3 Optimal Monetary Policy

When we add up expected utilities across agents in a stationary equilibrium, our welfare measure is

$$W = \rho\{u(x_1) - x_1\} + (1 - \rho)\{\theta u(x_2) - x_2\} \quad (2.56)$$

that represents the sum of the trade surpluses in the *DM*. Note that the first-best is (x_1^*, x_2^*) where $x_1^* > x_2^*$ and marginal utility of each type buyer is same when the allocations are along the curve with $u'(x_1) = \theta u'(x_2)$.

In order to know whether the liquidity-trap equilibrium is optimal, I consider the optimal monetary policy in a neighborhood of the equilibrium allocation with $z = \theta$ since the consumption level of liquidity-trap equilibrium would be greater as long as $\theta < 1$. Since the welfare function and feasible allocations in the (2.49) can be described in a (x_1, x_2) plain, I

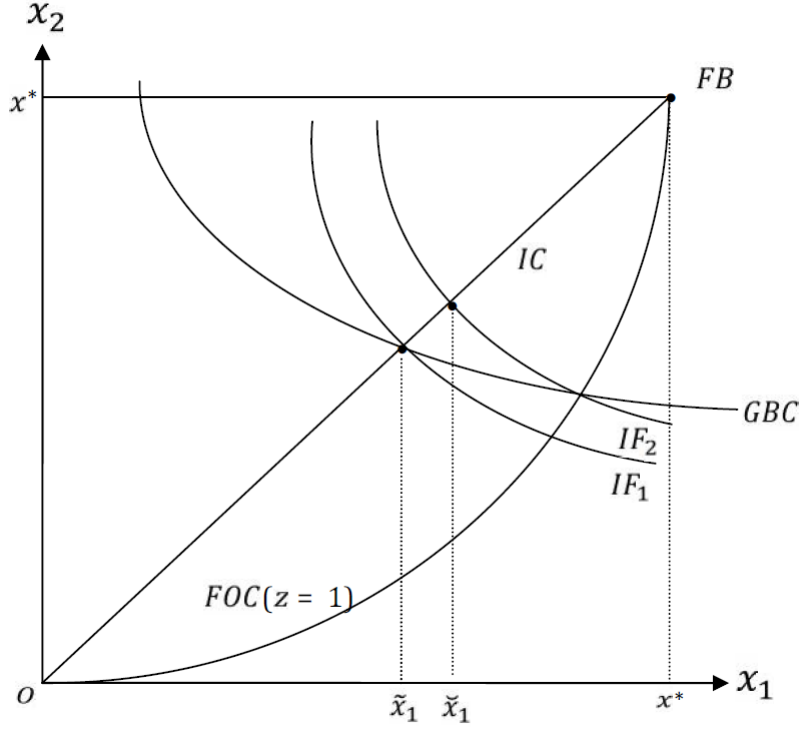


Figure 2.7: Optimal Monetary Policy

compare their slopes at $x_1 = \tilde{x}_1$.

At $x_1 = \tilde{x}_1$ the allocation satisfies with binding incentive constraint, $x = x_1 = x_2$. Thus in the (x_1, x_2) plane the slope of welfare function at $x = x_1 = x_2$ is

$$\frac{\partial x_2}{\partial x_1} = -\frac{\rho\{u'(x) - 1\}}{(1 - \rho)\{\theta u'(x) - 1\}} < -\frac{\rho}{(1 - \rho)\theta}. \quad (2.57)$$

The slope of the government budget constraint (2.49) at $x = x_1 = x_2$ is

$$\frac{\partial x_2}{\partial x_1} = -\frac{\rho\{u'(x) + xu''(x)\}}{(1 - \rho)\theta\{u'(x) + xu''(x)\} - K'(x_2)} > -\frac{\rho}{(1 - \rho)\theta} \quad (2.58)$$

where $K(x_2) = \frac{\beta y^i \theta u'(x_2)}{1 - \beta \theta u'(x_2)}$. Since $u'(x) + xu''(x) > 0$ and $K'(x_2) < 0$, the slope of welfare function is steeper at liquidity trap equilibrium. It implies that the optimal equilibrium allocation is achieved at the *Liquidity-trap equilibrium*.

2.5 Discussion

It is certain that the liquidity trap - a situation in which the implementation of monetary policy cannot influence in the market and real economy - is a serious concern for policy makers. In the history of the Great Depression the short-term interest rates decreased to zero in 1930-1932 and remained at zero for several years. Excess reserves in the banks increased and bank credit failed to expand until 1936-1937. Brunner and Meltzer (1968) suggest as an alternative that a trap could have been operated within the banking system when banks desired to hold excess reserves and were unwilling to lend. It is also shown that the Federal Reserve bank officers had this situation in mind: Chairman Eccles testified at the U.S. Congress in 1935 that, even if currency was used to purchase government bonds from the public, there would be no increase in the money supply or in bank credit.

Mr. Cross: “Why not pay off all government bonds and get rid of paying any interest—because that would be inflation itself?”

Governor Eccles: “Here is what would happen... such-action would simply increase the reserves of the banking system by the amount of government bonds which were purchased with currency. The currency would go out, if it was \$10 billion or \$20 billion or \$3 billion, whatever amount the government paid out in currency to retire its bonds; but the currency would immediately go into the banks and from the banks into the Federal Reserve banks... and you would have; additional reserves, additional excess reserves...”

It is hard to confirm that there was a liquidity trap in the Great Depression and that this liquidity trap occurred because banks desired to hold excess reserves. However, this paper shows that there is a possibility of a liquidity trap when banks have an incentive to hold liquid assets in their balance sheet.

2.6 Conclusion

In the paper I construct a banking model to study how private information confines liquidity insurance and the implementation of monetary policy. Given idiosyncratic liquidity shocks, lack of memory generates private information on types. A truth-telling banking contract is offered to provide liquidity efficiently under private information. When the supply of total assets is not enough to support liquidity distribution, the truth-telling incentive constraint binds and a liquidity premium arises in the price of illiquid assets. In the extended model with monetary policy when the truth-telling constraint binds, there exists a liquidity trap in which open market operations are ineffective in real allocations. This *liquidity-trap equilibrium* is different from the previous ones with currency-shortage or zero-lower-bound because it is generated by the incentive of banks to hold illiquid and even liquid assets for efficient liquidity provision.

This paper takes a step forward to understand the liquidity trap. It provides a model in which the liquidity trap can exist when banks have an incentive to hold liquid assets in their balance sheets so that it opens a possibility of studying the liquidity trap further. But it also leaves further questions unanswered. For example, liquid assets in a bank's balance sheet can play a role in preventing bank runs. In this respect we can ask how fragility of banks is associated with the effectiveness of monetary policy. This question requires a deeper consideration and explicit modeling on bank runs as a part of the model for the liquidity trap.

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